Mesh-based Impedance Sensitivity Formulation for DC/AC Power Integrity Design and Diagnosis

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***Abstract*—Accurate Power Distribution Network (PDN) design is crucial for Signal/Power Integrity (SI/PI) and Electromagnetic Interference (EMI) compliance. Achieving target power-ground (PG) noise levels for low power complex PDNs requires several design and analysis cycles. Although several classes of analysis tools, 2.5D and 3D, are commercially available, the presence of design tools are limited e.g. parametric design space exploration using multiple forward analysis. In this work, a frequency domain mesh-based sensitivity formulation for DC and AC impedance of PDNs is proposed. The two main objectives include: (i) highlighting layout regions to the designer for maximum impact in achieving target specifications and (ii) predicting the results of a design variant with mesh-based sensitivity information from the base-design. The time required for updating the results for the design variant is negligible compared to a complete re-simulation.**

Keywords— Power Integrity, 2.5D, Sensitivity, Non-orthogonal PEEC

I. Introduction

PDNs are designed to supply noise free regulated voltages to different sections of the chip. With decreasing voltage levels for low-power applications and demand for higher drive current for increased functionality, the design of optimal PDNs has become increasingly challenging. A typical design cycle consists of several design iterations involving changes in layout, decoupling capacitor placement and values etc. An electromagnetic (EM) simulation tool is required for the analysis of each design variant. 2.5D based tools e.g. Multi-Layered Finite Difference Method (MFDM) [1], 2.5D PEEC method [2] etc. have been used in the past for power-ground modeling and preferred to 3D full-wave EM analysis due to computational efficiency. However, in a design cycle, the existing commercial and academic EM solver tools treat each design variant as an independent simulation. The changes in the design layout across design iterations to achieve target specifications, is currently driven primarily by designer intuition alone. To that end, the availability of PDN design specific tools is limited.

Design space exploration techniques have been developed in the past for individual PDN components like vias, slots, decoupling caps. In [3], a method of sensitivity computation of PDN impedance with decoupling capacitors is presented for MFDM. In [4] analytical expressions for multi-parametric sensitivity analysis of S-Parameters of coupled vias is presented. Parametric sweeps, Response Surface Modeling (RSM), monte-carlo or Design of Experiment (DoE) based techniques require several number of forward solves to present the designer with a view of the sensitivity of chosen parameters to the impedance/s-parameters. Further, for irregular shapes, defining parameters in design optimization restricts the designer's freedom.

In this work, a mesh-based sensitivity approach is proposed as an effective design-aid for PDN design. By plotting the magnitude of the sensitivity over all mesh elements of the PDN structure, with respect to a desired performance parameter, the designer can easily diagnose culprits for poor electrical performance and consequently regions of modification for the next design iteration. The analytical sensitivity can be used for a fast design update, allowing calculation of PDN impedance of design variants with a single forward solve. To achieve this goal, a non-orthogonal mesh is required. The shape of each quadrilateral can be modified to capture small design variations effectively keeping the same number of mesh elements, which is not possible using a rectangular grid. Therefore, the 2.5D PEEC formulation [2] is used as the basic EM solver. The sensitivity of R, L, G, C parameters to mesh elements is computed analytically and plugged in a SPICE-like formulation to get the sensitivity of port electrical parameters to every mesh element. The Adjoint Variable Method (AVM) [5] facilitates computation of the impedance sensitivity to every mesh element with a single forward solve. Analytically obtaining sensitivity as opposed to the finite difference method prevents inaccuracies due to large step size, instability due to small step sizes and freedom from time consuming multiple forward solutions. The main contribution of this work is (i) to provide an analytical expression for mesh-based impedance sensitivity (ii) to predict the impedance of incremental designs based on this sensitivity.

II. AC/DC Power Integrity Analysis

A. Resistance calculation using non-orthogonal 3D PEEC

A unit non-orthogonal PEEC cell can be found in [6] but it is also shown in Fig.1 for ease of explanation in this paper.

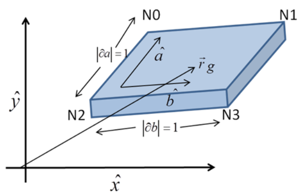


Fig. 1. Unit non-orthogonal PEEC quadrilateral cell

Here, $\vec{r_g}$ represents the quadrilateral in global coordinates:

$$\frac{\partial \vec{r}_g}{\partial a} = \frac{\partial x}{\partial a}\hat{x} + \frac{\partial y}{\partial a}\hat{y} \qquad \frac{\partial \vec{r}_g}{\partial b} = \frac{\partial x}{\partial b}\hat{x} + \frac{\partial y}{\partial b}\hat{y} \tag{1}$$

and $\vec{a}, \vec{b}$ are the local co-ordinate system unit vectors given as

$$\vec{a} = \frac{\partial \vec{r_g}}{\partial a} \Big/ \left|\frac{\partial \vec{r_g}}{\partial a}\right| \qquad \vec{b} = \frac{\partial \vec{r_g}}{\partial b} \Big/ \left|\frac{\partial \vec{r_g}}{\partial b}\right|. \tag{2}$$

For ease of representation in this work, the following vectors are renamed as:

$$\vec{k} = \frac{\partial \vec{r}g}{\partial a} \qquad \vec{l} = \frac{\partial \vec{r}g}{\partial b} \tag{3}$$

$\vec{k}, \vec{l}$, describe the quadrilateral shape. If ρ and t are the conductor resistivity and thickness respectively:

$$Ra = \frac{\rho}{t} \times \int_{-1}^{1}\int_{-1}^{1} \frac{|\vec{k}|^2}{|\vec{k} \times \vec{l}|} dadb \qquad Rb = \frac{\rho}{t} \times \int_{-1}^{1}\int_{-1}^{1} \frac{|\vec{l}|^2}{|\vec{l} \times \vec{k}|} dadb \tag{4}$$

B. R,L,G,C calculation for 2.5D PEEC

In 2.5D PEEC, every mesh element has a ground reference element of identical shape. Details of the derivation of R, L, G, C can be found in [2]. If d is the spacing between two plates, ϵ and μ are the permittivity and permeability, $tan\delta$ the loss tangent of the material then resistance is computed as twice as in (4) and L, G and C parameters are given by:

$$La = \mu d \times \int_{-1}^{1}\int_{-1}^{1} \frac{|\vec{k}|^2}{|\vec{l} \times \vec{k}|} dadb \qquad Lb = \mu d \times \int_{-1}^{1}\int_{-1}^{1} \frac{|\vec{l}|^2}{|\vec{k} \times \vec{l}|} dadb \tag{5}$$

$$C = \frac{\varepsilon}{d} \times \int_{-1}^{1}\int_{-1}^{1} |\vec{k} \times \vec{l}| dadb \qquad G = \omega C \tan\delta \tag{6}$$

This formulation is benchmarked with respect to a commercial tool as shown in Fig. 2.

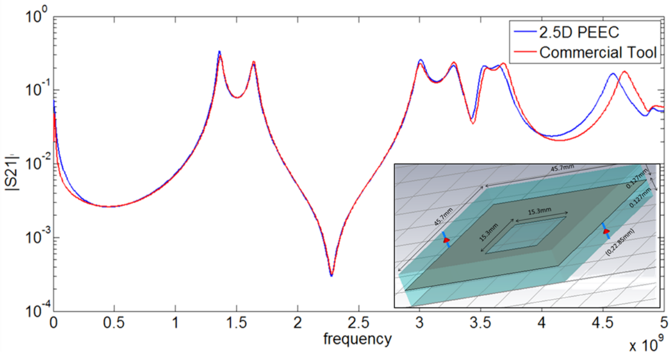


Fig. 2. Plot of |S21| simulated with in-house solver and commercial tool. The structure is shown as inset.

III. Mesh-Based Sensitivity of Z

The sensitivity of a function of several design variables is the derivative of that function in the design variable space. In this work, the target function explored is the Z parameters of a multi-port structure and the design variables are the individual mesh shape variables, defined in (3) as $\vec{k}, \vec{l}$. AVM is an efficient method to calculate sensitivity of functions that require solution of large matrices for every design variable combination. A detailed explanation of AVM and it's applicability to EM design optimization is given in [5]. AVM is a fast approach to finding

$$\nabla_{\mathbf{x}}\text{f, subject to} \boldsymbol{YV} = \boldsymbol{I}\text{, where} \nabla_{\mathbf{x}} = [\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \dots \frac{\partial}{\partial x_M}] \tag{7}$$

$$\nabla_{\mathbf{x}}\text{f} = \nabla_{\mathbf{x}}^{\mathbf{e}}\text{f} + (\nabla_{\mathbf{V}}\text{f}(\boldsymbol{Y})^{-1})(\nabla_{\mathbf{x}}\mathbf{I} - \nabla_{\mathbf{x}}\mathbf{YV}) \tag{8}$$

For this work, x is the set$[\vec{k}, \vec{l}]$, $f = \frac{v_{port}}{i_{port}}$, $\mathbf{V}$ is the solution to the initial design in the form of node-voltages and $\mathbf{I}$ is array of source currents at each node in the mesh. If $i_{port} = 1$ then $\mathbf{I}$ is an array with one non-zero element and $f = v_{port}$, making $\nabla_{\mathbf{V}}\text{f}$ an array with only one non-zero element. $\nabla_{\mathbf{x}}^{\mathbf{e}}\text{f} = 0$ and $\nabla_{\mathbf{x}}\mathbf{I} = 0$ due to lack of dependency on the design variables. The calculation of $\nabla_{\mathbf{V}}\text{f}(\boldsymbol{Y})^{-1}$ and $\mathbf{V}$ require a single forward solution of the initial design. The sensitivity of Z with every mesh element can be obtained by a sparse array multiplication of $\nabla_{\mathbf{x}}\mathbf{YV}$ that changes for every x and $\nabla_{\mathbf{V}}\text{f}(\boldsymbol{Y})^{-1}$, that is only dependent on the initial solution.

The computation of $\nabla_{\mathbf{x}}\mathbf{Y}$ is explained in this section. The network of RLGC for a structure can be analyzed by a SPICE formulation. The representative Modified Nodal Analysis (MNA) matrix $(\mathbf{Y})$ of an individual mesh quadrilaterals is given by:

$$\begin{bmatrix} (G_1 + j\omega C_1) & (G_2 + j\omega C_2) & (-G_1 - j\omega C_1) & (-G_2 - j\omega C_2) & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ (-G_1 - j\omega C_1) & 0 & (G_1 + j\omega C_1) & 0 & 0 & 1 \\ (-G_2 - j\omega C_2) & 0 & 0 & (G_2 + j\omega C_2) & 0 & -1 \\ 1 & -1 & 0 & 0 & -j\omega L_{a1} + R_1 & -j\omega M_2 \\ 0 & 0 & 1 & -1 & -j\omega L_{b1} + R_2 & -j\omega M_1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ I_{L1} \\ I_{L2} \end{bmatrix} = \begin{bmatrix} Is_1 \\ Is_2 \\ Is_3 \\ Is_4 \\ Vs_1 \\ Vs_2 \end{bmatrix} \tag{9}$$

As $\vec{k}, \vec{l}$ are each vectors in two dimensions x, y, the formulae below are written in terms of k_x. It should be kept in mind that shape of each mesh element is represented as $[k_x k_y l_x, l_y]$.

$$\frac{\partial Y}{\partial k_x} = \begin{bmatrix} (\frac{\partial G_1}{\partial k_x} + j\omega\frac{\partial C_1}{\partial k_x}) & (\frac{\partial G_2}{\partial k_x} + j\omega\frac{\partial C_2}{\partial k_x}) & (-\frac{\partial G_1}{\partial k_x} - j\omega\frac{\partial C_1}{\partial k_x}) & (-\frac{\partial G_2}{\partial k_x} - j\omega\frac{\partial C_2}{\partial k_x}) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ (-\frac{\partial G_1}{\partial k_x} - j\omega\frac{\partial C_1}{\partial k_x}) & 0 & (\frac{\partial G_1}{\partial k_x} + j\omega\frac{\partial C_1}{\partial k_x}) & 0 & 0 & 0 \\ (-\frac{\partial G_2}{\partial k_x} - j\omega\frac{\partial C_2}{\partial k_x}) & 0 & 0 & (\frac{\partial G_2}{\partial k_x} + j\omega\frac{\partial C_2}{\partial k_x}) & 0 & 0 \\ 0 & 0 & 0 & 0 & -j\omega\frac{\partial L_1}{\partial k_x} + \frac{\partial R_1}{\partial k_x} & -j\omega\frac{\partial L_2}{\partial k_x} \\ 0 & 0 & 0 & 0 & -j\omega\frac{\partial L_2}{\partial k_x} + \frac{\partial R_2}{\partial k_x} & -j\omega\frac{\partial L_1}{\partial k_x} \end{bmatrix} \tag{10}$$

Expressions for RLGC sensitivities for k_x are shown below, the ones for k_y, l_x, l_y can be obtained similarly.

$$\frac{\partial Ra}{\partial k_x} = \frac{2\rho}{t} \times \int_{-1}^{1}\int_{-1}^{1} \frac{2k_x(k_x l_y - k_y l_x)^2 - l_y((k_x{}^2 + k_y{}^2)(k_x l_y - k_y l_x))}{(k_x l_y - k_y l_x)^2 |(k_x l_y - k_y l_x)|} dadb \tag{11}$$

$$\frac{\partial Rb}{\partial k_x} = \frac{2\rho}{t} \times \int_{-1}^{1}\int_{-1}^{1} \frac{l_y((l_x{}^2 + l_y{}^2)(l_x k_y - l_y k_x))}{(l_x k_y - l_y k_x)^2 |(l_x k_y - l_y k_x)|} dadb \tag{12}$$

$$\frac{\partial La}{\partial k_x} = \mu d \times \int_{-1}^{1}\int_{-1}^{1} \frac{2k_x(l_x k_y - l_y k_x)^2 + l_y((k_x{}^2 + k_y{}^2)(l_x k_y - l_y k_x))}{(l_x k_y - l_y k_x)^2 |(l_x k_y - l_y k_x)|} dadb \tag{13}$$

$$\frac{\partial Lb}{\partial k_x} = \mu d \times \int_{-1}^{1}\int_{-1}^{1} -\frac{l_y((l_x{}^2 + l_y{}^2)(k_x l_y - k_y l_x))}{(k_x l_y - k_y l_x)^2 |(k_x l_y - k_y l_x)|} dadb \tag{14}$$

$$\frac{\partial C}{\partial k_x} = \frac{\varepsilon}{d} \times \int_{-1}^{1}\int_{-1}^{1} \frac{l_y(k_x l_y - k_y l_x)}{|(k_x l_y - k_y l_x)|} dadb \qquad \frac{\partial G}{\partial k_x} = \omega \tan\delta \frac{\partial C}{\partial k_x} \tag{15}$$

Once the sensitivity to all the mesh elements is obtained, the impedance of a design variant achieved by modification of the same mesh is given by:

$$Z_{new} = Z_{old} + \sum_{i=1}^{M} \frac{\partial Z}{\partial k_{xi}(l_{xi})} \Delta k_{xi}(l_{xi}) + \sum_{i=1}^{M} \frac{\partial Z}{\partial k_{yi}(l_{yi})} \Delta k_{yi}(l_{yi}) \quad (16)$$

IV. Numerical Results

A. Case 1: (DC) Sensitivity of R with mesh

A power plane with a 1A current source at input port, as shown in Fig. 3, is used for analysis here. The magnitude of the mesh-based sensitivities in log scale are plotted in Fig. 3.

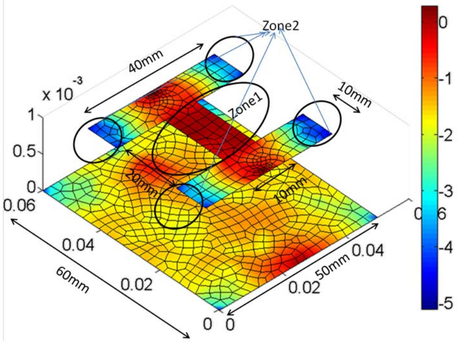
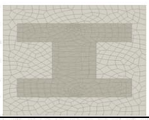
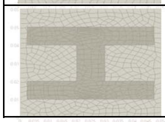


Fig. 3. Mesh-based sensitivity magnitude plotted for a power-ground structure

The plot indicates that any change in the blue colored areas, like Zone 2 will have little impact on the R value whereas any change in the dark red areas like Zone 1 will have a large impact on R. The pin sizes are also critical. Two design variants, along with their simulated and predicted values of R, with an increase in Zone 1 and 2 are shown in Table I.

TABLE I: Impact of Design Variation on R and Prediction Accuracy

	Design	Simulated R	Predicted R from D0
	D0	0.1209 Ω	-
	D1	0.1086 Ω	0.1049 Ω
	D2	0.1224 Ω	0.1224 Ω

B. Case 2: (AC) Sensitivity of Loop Inductance with mesh

Loop inductance is a major design consideration for decoupling capacitor placement. In this case study, a rectangular copper plate with a slot on top of ground plane is analyzed using a 2.5D formulation for loop inductance sensitivity at 1.5GHz as shown in Fig. 4.

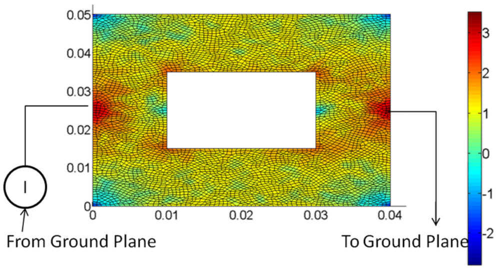


Fig. 4. Sensitivity of loop-inductance to mesh

The plot indicates that apart from the pin-size, the loop inductance is more sensitive to variations of structure around the edges in the Y-direction as compared to geometry variation between the source and ground nodes. This was tested by simulating two design variants at 1.5GHz. The nominal design with geometry shown in Fig. 4 has a loop inductance of 4.9nH. By decreasing the slot width to 10mm, inductance reduces to 4.4nH and by decreasing slot length to 10mm, inductance drops to 4.2nH. As can be seen, the change in geometry in X-direction has lower impact as opposed to change in geometry in the Y-direction.

C. Case 3: (AC) Impedance curve update for design-variant

This case explores the ability of the mesh-based sensitivity update method to correctly predict the impedance curve in frequency domain for a design-variant using sensitivity information from the solution of the base design without any further re-simulation. Initial design is a copper plate of 10um thickness separated from ground with dielectric of 1mm thickness and $\epsilon = 4, \mu = 1, tan\delta = 0.02$ shown as D0 right-inset in Fig. 5. For modification, the length of the hole is increased, shown as D2, right-inset in Fig. 5. The impedance profile is simulated over a frequency range and plotted for initial and modified designs. Also plotted is the impedance predicted from the initial design using sensitivity based update.

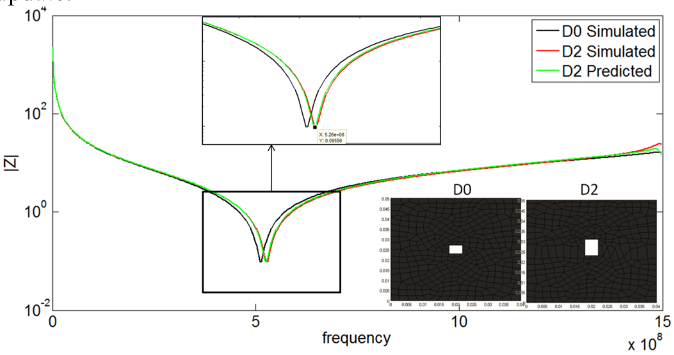


Fig. 5. Sensitivity based prediction accuracy for design variant D2 from base-design D0

Reference

[1] A. E. Engin, K. Bharath and M. Swaminathan, "Multilayered Finite-Difference Method (MFDM) for Modeling of Package and Printed Circuit Board Planes," *IEEE Transactions on Electromagnetic Compatability*, vol. 49, no. 2, pp. 441-447, May 2007.

[2] B. Prasad Nayak , S. R.Vedicherla, and Dipanjan Gope, "Non-orthogonal 2.5D PEEC for Power Integrity Analysis," in *IEEE Electrical Design of Advanced Packaging and Systems Symposium (EDAPS), 2015* , Seoul, 2015, pp. 174 - 177.

[3] A. E. Engin, "Efficient Sensitivity Calculations for Optimization of Power Delivery Network Impedance," *IEEE Transactions on Electromagnetic Compatability*, vol. 52, no. 2, pp. 332-339, May 2010.

[4] Y. Li and V. Jandhyala, "Multiparameter Sensitivity Analysis of Multiple Coupled Vias in Board and Package Structures for Early Design and Optimization," *IEEE Transactions on Advanced Packaging*, vol. 33, no. 4, pp. 1003-1011, November 2010.

[5] N. K. Georgieva, S. Glavic, M. H. Bakr and J. W. Bandler, "Feasible Adjoint Sensitivity Technique for EM Design Optimization," *IEEE Transactions on Microwave Theory and Techniques*, vol. 50, no. 12, pp. 2751-2757, December 2002.

[6] A. E. Ruehli , G. Antonini, J. Esch, J. Ekman, A. Mayo and A. Orlandi, "Nonorthogonal PEEC Formulation for Time and Frequency-Domain EM and Circuit Modeling," *IEEE Transactions on Electromagnetic Compatibility*, vol. 45, no. 2, pp. 167-176, May 2003.