Modeling and Simulation of High Speed I/O Links Using X Parameters

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***Abstract*—In this work, the polyharmonic distortion (PHD) model is used to analyze the behavior of high-speed nonlinear links. The model assumes the validity of the harmonic superposition principle for high-speed I/O links. From the PHD formalism, a frequency-domain X-parameter matrix formulation is derived. The formulation accommodates both port and harmonic dependence of the signals. Relationships are derived that permit to combined various nonlinear sub-blocks described by their X parameters. Simulations are performed and compared with other methods.**

Keywords-Nonlinear vector network analyzer, polyharmonic distortion, scattering waves, X-parameters.

I. INTRODUCTION

Modern microprocessors and communication networks rely on complex multicore streaming architectures to achieve high levels of performance. As data rates keep increasing to multi gigabits per second, complex interactions between channel links, integrated circuits, packages and power distribution networks take place and become more difficult to predict and manage.

Simulation techniques for the prediction of signal propagation in these environments have been the subject of investigative research for the past decade. Behavioral and macro-modeling techniques have been among the most popular methods used to predict the performance of these systems.

One major challenge in the prediction of waveforms propagating in high-speed links is the nonlinear behavior of the active building blocks. These blocks may include I/O buffer as well as equalizer systems that aim at improving the signal integrity in these systems. In order to handle nonlinearities, most simulation techniques have been performed in the time domain using convolution-based techniques.

The introduction of X parameters by Verspecht and Root [1], [2] has opened up new possibilities in the arena of nonlinear electronic computer-aided design. Using the polyharmonic distortion (PHD) method, a mathematically robust framework can be implemented and used to construct accurate representations for electronic systems that include nonlinearities. Based on the harmonic superposition principle, the X parameter formalism builds on scattering parameter theory and for which it is a superset. Relationship between incident and scattered waves are described using not only port-to-port but also harmonic-to-harmonic interactions.

In this paper, we employ the PHD formalism to construct a general representation for I/O channels that accounts for both linear and nonlinear elements. We first review the fundamentals of X-parameters; next we develop a formulation for a multiport network that is then used to model high-speed links. Frequency and time-domain results are analyzed and the proposed method is compared with results from other techniques.

II. X-PARAMETER FORMALISM

The PHD formulation builds from the representation of independent and dependent power waves associated with a network in terms of its scattering parameters. Consider an n-port network; in the PHD method, a relationship between the incident and scattered waves can be expressed as:

$$b_p^{(k)} = \sum_{q,l} S_{pq}^{(kl)} a_q^{(l)} P^{k+l} + T_{pq}^{(kl)} a_q^{(l)*} P^{k-l} \tag{1}$$

where $a_q^{(l)}$ is the l^{th} harmonic of the incident wave into port q - subscripts indicate port interaction while superscripts within parentheses indicate harmonic interaction - $b_p^{(k)}$ is the k^{th} harmonic of the scattered wave at port p. $S_{pq}^{(kl)}$ is a scattering parameter of type S that accounts for the contribution to the k^{th} harmonic at port p due to the l^{th} harmonic of the incident wave in port q. $T_{pq}^{(kl)}$ is a scattering parameter of type T that accounts for the contribution to the k^{th} harmonic at port p due to the l^{th} harmonic of the conjugate of the incident wave in port q. The existence of a scattering parameter of type T is due to the nonanalyticity of the spectral mapping from the time domain to the frequency domain [2]; in essence, it emphasizes the fact that unlike in a linear system, the real and imaginary parts of the waves in a nonlinear system are treated differently through their transfer functions. P is a phase shift term which must be accounted for during the interactions between the various harmonics. By convention, P is chosen to be associated with the phase of $a_1^{(1)}$

$$P = e^{j2\pi(Arg\{a_1^{(1)}\})} \tag{2}$$

The S's and T's represent a complete set of network parameters that fully characterize the reactive, nonlinear dynamics of the network under study. These parameters are obtained by first applying a large-signal stimulus at the fundamental frequency at the reference port; additional stimuli at the various harmonics are then superimposed with and without phase shift at all the ports to produce the corresponding responses from which parameters of the two types are extracted [2]. This operation can be performed using circuit simulation techniques. Nonlinear vector network analyzers (NVNA) are also available that will perform the measurements [3]-[6].

III. MATRIX REPRESENTATION

The presence of the scattering parameter of type T multiplying the conjugate of the waves motivates the need for special manipulation of the scattering parameter coefficients. Without loss of generality, we can temporarily adopt a simpler notation in which subscripts and superscripts are omitted that reads

$$b = Sa + Ta^{*} \tag{3}$$

in which a and b are the incident and reflected signals associated with arbitrary input and output ports and harmonics. S and T are the scattering parameters of type S and T respectively. We can split the wave variables and network coefficients into real and imaginary components to read

$$b_r + jb_i = \left(S_r + jS_i\right)\left(a_r + ja_i\right) + \left(T_r + jT_i\right)\left(a_r - ja_i\right), \tag{4}$$

where the subscripts r and i refer to real and imaginary parts respectively. After rearranging and separating the variables into their real and imaginary components we get:

$$b_r = \left(S_r + T_r\right)a_r - \left(S_i - T_i\right)a_i, \tag{5}$$

$$b_i = \left(S_i + T_i\right)a_r + \left(S_r - T_r\right)a_i \tag{6}$$

This can be arranged in a matrix form as

$$\begin{pmatrix} b_r \\ b_i \end{pmatrix} = \begin{pmatrix} X_{rr} & X_{ri} \\ X_{ir} & X_{ii} \end{pmatrix} \begin{pmatrix} a_r \\ a_i \end{pmatrix} \tag{7}$$

where

$$X_{rr} = \left(S_r + T_r\right),\ \ X_{ri} = -\left(S_i - T_i\right), \tag{8}$$

$$X_{ir} = \left(S_i + T_i\right),\ \ X_{ii} = \left(S_r - T_r\right) \tag{9}$$

Matrix equation (7) is an expression of the non-analytic nature of the spectral mapping function described by the X parameter formalism. It emphasizes that the real and imaginary parts of the waves are treated differently in a manner that cannot be described by a complex product. Consequently, an X-parameter matrix formulation must separate not only harmonics and ports but also real and imaginary components of the parameters and associated signals. For analysis purposes, the phase-normalized version (P=1) of equation (1) is utilized for describing the wave interactions. Referring to the network diagram in Figure 1, for which we assume n ports and m harmonics, we can define incident and reflected wave vectors given by

$$\mathbf{a} = \begin{pmatrix} \mathbf{a_1} \\ \mathbf{a_2} \\ . \\ \mathbf{a_n} \end{pmatrix}, \ \text{and}\ \ \mathbf{b} = \begin{pmatrix} \mathbf{b_1} \\ \mathbf{b_2} \\ . \\ \mathbf{b_n} \end{pmatrix}, \tag{10}$$

where each of the $\mathbf{a_p}$'s and $\mathbf{b_p}$'s are subvector representing incident and reflected signals at port p respectively. More explicitly:

$$\mathbf{a_p} = \begin{pmatrix} a_{pr}^{(1)} \\ a_{pi}^{(1)} \\ a_{pr}^{(2)} \\ a_{pi}^{(2)} \\ . \\ . \\ a_{pr}^{(m)} \\ a_{pi}^{(m)} \end{pmatrix}, \ \text{and}\ \ \mathbf{b_p} = \begin{pmatrix} b_{pr}^{(1)} \\ b_{pi}^{(1)} \\ b_{pr}^{(2)} \\ b_{pi}^{(2)} \\ . \\ . \\ b_{pr}^{(m)} \\ b_{pi}^{(m)} \end{pmatrix}, \tag{11}$$

where $a_{pr}^{(k)}$ and $b_{pr}^{(k)}$ are the real parts of the k^{th} harmonic of the incident and reflected waves, respectively. From these relationships, it is clear that an X-parameter matrix can be defined to relate these waves in the form **b=Xa** where

$$\mathbf{X} = \begin{pmatrix} \mathbf{X_{11}} & \mathbf{X_{12}} & . & \mathbf{X_{1n}} \\ \mathbf{X_{21}} & \mathbf{X_{22}} & . & . \\ . & . & . & . \\ \mathbf{X_{n1}} & . & . & \mathbf{X_{nn}} \end{pmatrix} \tag{12}$$

in which each $\mathbf{X_{pq}}$ is a submatrix (size $2m \times 2m$) given by

$$\mathbf{X_{pq}} = \begin{pmatrix} X_{pqrr}^{(11)} & X_{pqri}^{(11)} & X_{pqrr}^{(12)} & X_{pqri}^{(12)} & . & . & X_{pqrr}^{(1m)} & X_{pqri}^{(1m)} \\ X_{pqir}^{(11)} & X_{pqii}^{(11)} & X_{pqir}^{(12)} & X_{pqii}^{(12)} & . & . & . & . \\ X_{11rr}^{(21)} & X_{11ri}^{(21)} & X_{11rr}^{(22)} & X_{pqri}^{(22)} & . & . & . & . \\ X_{pqir}^{(21)} & X_{pqii}^{(21)} & X_{pqir}^{(22)} & X_{pqii}^{(22)} & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ X_{pqir}^{(m1)} & X_{pqii}^{(m1)} & . & . & . & . & X_{pqir}^{(mm)} & X_{pqii}^{(mm)} \end{pmatrix} \tag{13}$$

for which we made use of the relationship in (8)-(9). More explicitly,

$$X_{pqrr}^{(kl)} = \mathrm{Re}\left[S_{pq}^{(kl)}\right] + \mathrm{Re}\left[T_{pq}^{(kl)}\right] \tag{14}$$

$$X_{pqri}^{(kl)} = \mathrm{Im}\left[T_{pq}^{(kl)}\right] - \mathrm{Im}\left[S_{pq}^{(kl)}\right] \tag{15}$$

$$X_{pqir}^{(kl)} = \mathrm{Im}\left[S_{pq}^{(kl)}\right] + \mathrm{Im}\left[T_{pq}^{(kl)}\right], \quad (16)$$

$$X_{pqii}^{(kl)} = \mathrm{Re}\left[S_{pq}^{(kl)}\right] - \mathrm{Re}\left[T_{pq}^{(kl)}\right]. \quad (17)$$

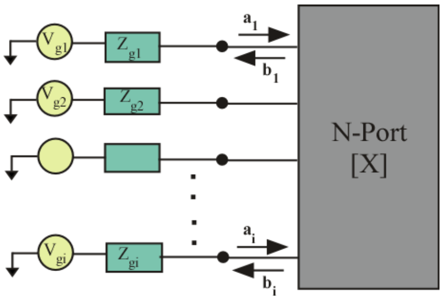

Figure 1. N-port described by X parameters with termination and source.

As an example, $X_{pqri}^{(kl)}$ is the contribution to the real part of the k^{th} harmonic of the wave scattered at the p^{th} port due to the imaginary part of the l^{th} harmonic of the incident port at the q^{th} port. The complete X matrix is of size $2mn \times 2mn$. For instance in the case of a two-port network and taking into account 2 harmonics (fundamental and second harmonic), the full vector and matrix description would be

$$\mathbf{a} = \begin{pmatrix} a_{1r}^{(1)} \\ a_{1i}^{(1)} \\ a_{1r}^{(2)} \\ a_{1i}^{(2)} \\ a_{2r}^{(1)} \\ a_{2i}^{(1)} \\ a_{2r}^{(2)} \\ a_{2i}^{(2)} \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} b_{1r}^{(1)} \\ b_{1i}^{(1)} \\ b_{1r}^{(2)} \\ b_{1i}^{(2)} \\ b_{2r}^{(1)} \\ b_{2i}^{(1)} \\ b_{2r}^{(2)} \\ b_{2i}^{(2)} \end{pmatrix} \quad (18)$$

and

$$\mathbf{X} = \begin{pmatrix}
X_{11rr}^{(11)} & X_{11ri}^{(11)} & X_{11rr}^{(12)} & X_{11ri}^{(12)} & X_{12rr}^{(11)} & X_{12ri}^{(11)} & X_{12rr}^{(12)} & X_{12ri}^{(12)} \\
X_{11ir}^{(11)} & X_{11ii}^{(11)} & X_{11ir}^{(12)} & X_{11ii}^{(12)} & X_{12ir}^{(11)} & X_{12ii}^{(11)} & X_{12ir}^{(12)} & X_{12ii}^{(12)} \\
X_{11rr}^{(21)} & X_{11ri}^{(21)} & X_{11rr}^{(22)} & X_{11ri}^{(22)} & X_{12rr}^{(21)} & X_{12ri}^{(21)} & X_{12rr}^{(22)} & X_{12ri}^{(22)} \\
X_{11ir}^{(21)} & X_{11ii}^{(21)} & X_{11ir}^{(22)} & X_{11ii}^{(22)} & X_{12ir}^{(21)} & X_{12ii}^{(21)} & X_{12ir}^{(22)} & X_{12ii}^{(22)} \\
X_{21rr}^{(11)} & X_{21ri}^{(11)} & X_{21rr}^{(12)} & X_{21ri}^{(12)} & X_{22rr}^{(11)} & X_{22ri}^{(11)} & X_{22rr}^{(12)} & X_{22ri}^{(12)} \\
X_{21ir}^{(11)} & X_{21ii}^{(11)} & X_{21ir}^{(12)} & X_{21ii}^{(12)} & X_{22ir}^{(11)} & X_{22ii}^{(11)} & X_{22ir}^{(12)} & X_{22ii}^{(12)} \\
X_{21rr}^{(21)} & X_{21ri}^{(21)} & X_{21rr}^{(22)} & X_{21ri}^{(22)} & X_{22rr}^{(21)} & X_{22ri}^{(21)} & X_{22rr}^{(22)} & X_{22ri}^{(22)} \\
X_{21ir}^{(21)} & X_{21ii}^{(21)} & X_{21ir}^{(22)} & X_{21ii}^{(22)} & X_{22ir}^{(21)} & X_{22ii}^{(21)} & X_{22ir}^{(22)} & X_{22ii}^{(22)}
\end{pmatrix} \quad (19)$$

IV. Signal Simulation

In order to find a solution that incorporates the termination conditions, we start by stating a relationship between voltage waves and terminal variables. These variables satisfy the relationship

$$\mathbf{v} = \mathbf{a} + \mathbf{b} \quad (20)$$

and

$$\mathbf{i} = \mathbf{Z}_o^{-1}\left[\mathbf{a} - \mathbf{b}\right] \quad (21)$$

where $\mathbf{v}$ and $\mathbf{i}$ are the total voltage and current vectors respectively. $\mathbf{Z_o}$ is the reference impedance matrix constructed from the system used to determine the X-parameters of the black box. We next state the scattering voltage wave relationship as

$$\mathbf{b} = \mathbf{Xa}. \quad (22)$$

This must be combined with the termination conditions expressing the relationship between the voltage sources, terminal impedances, and voltage waves as

$$\mathbf{a} = \mathbf{Dv_s} + \mathbf{\Gamma b}. \quad (23)$$

$\mathbf{D}$ is a voltage division matrix, $\mathbf{\Gamma}$ is a reflection coefficient matrix and $\mathbf{v_s}$ is the voltage source vector. These matrices and vector are constructed in the same manner as those in (18) and (19). By combing (22) and (23), we get

$$\mathbf{a} = \left[\mathbf{1} - \mathbf{\Gamma X}\right]^{-1} \mathbf{Dv_s}, \quad (24)$$

which expresses the incident voltage wave vector in terms of the voltage source vector. The reflected voltage wave vector, **b** can be obtained from **a** using (22) . Then, the voltage and current solutions can be determined using (20) and (21).

V. Cascading Sub-networks

Most I/O link designs can be described as succession of two-port building blocks that facilitate the interfacing between, ICs, packages, connectors, and various types of interconnections and traces. These cascades may represent the succession between different domains of signaling (analog, digital, mixed-signal, and passive) as illustrated in Figure 2.

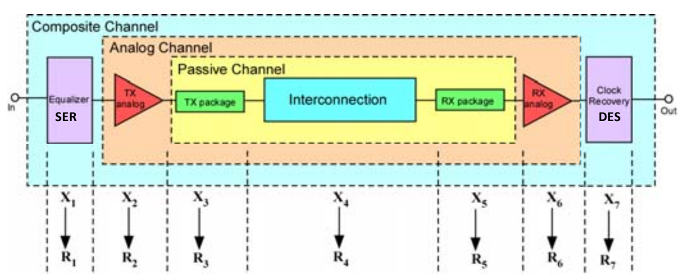

Figure 2. Generic diagram of high-speed link showing various building blocks.

When the blocks are described by their two-port X-parameter representation, a composite description of the system may be desired. For each two-port, **b=Xa** takes the form:

$$\begin{pmatrix} \mathbf{b_1} \\ \mathbf{b_2} \end{pmatrix} = \begin{pmatrix} \mathbf{X_{11}} & \mathbf{X_{12}} \\ \mathbf{X_{21}} & \mathbf{X_{22}} \end{pmatrix} \begin{pmatrix} \mathbf{a_1} \\ \mathbf{a_2} \end{pmatrix} \quad (25)$$

in which the sub-vectors and sub-matrices are defined as described in (10) and (11). From this relationship, a transfer matrix representation can be derived to take the form of

$$\begin{pmatrix} \mathbf{a}_1 \\ \mathbf{b}_1 \end{pmatrix} = \begin{pmatrix} \mathbf{R}_{aa} & \mathbf{R}_{ab} \\ \mathbf{R}_{ba} & \mathbf{R}_{bb} \end{pmatrix} \begin{pmatrix} \mathbf{a}_2 \\ \mathbf{b}_2 \end{pmatrix}, \tag{26}$$

where the sub-matrices are given by

$$\mathbf{R}_{aa} = -\mathbf{X}_{21}^{-1}\mathbf{X}_{22} \qquad \mathbf{R}_{ab} = \mathbf{X}_{21}^{-1} \tag{27}$$

$$\mathbf{R}_{ba} = \mathbf{X}_{12} - \mathbf{X}_{11}\mathbf{X}_{21}^{-1}\mathbf{X}_{22} \qquad \mathbf{R}_{bb} = \mathbf{X}_{11}\mathbf{X}_{21}^{-1}. \tag{28}$$

Conversely, the X-parameter sub-matrices can be recovered by using

$$\mathbf{X}_{11} = \mathbf{R}_{bb}\mathbf{R}_{ab}^{-1} \qquad \mathbf{X}_{12} = \mathbf{R}_{ba} - \mathbf{R}_{bb}\mathbf{R}_{ab}^{-1}\mathbf{R}_{aa} \tag{29}$$

$$\mathbf{X}_{21} = \mathbf{R}_{ab}^{-1} \qquad \mathbf{X}_{22} = -\mathbf{R}_{ab}^{-1}\mathbf{R}_{aa}. \tag{30}$$

If we use the notation

$$\mathbf{w} = \mathbf{R}\mathbf{u}, \tag{31}$$

where

$$\mathbf{R} = \begin{pmatrix} \mathbf{R}_{aa} & \mathbf{R}_{ab} \\ \mathbf{R}_{ba} & \mathbf{R}_{bb} \end{pmatrix}, \quad \mathbf{w} = \begin{pmatrix} \mathbf{a}_1 \\ \mathbf{b}_1 \end{pmatrix} \text{ and } \mathbf{u} = \begin{pmatrix} \mathbf{a}_2 \\ \mathbf{b}_2 \end{pmatrix}, \tag{32}$$

then it can be shown that the composite transfer matrix for a cascade of several two-port blocks is obtained by the successive multiplications of the transfer matrices, namely

$$\mathbf{R}_T = \mathbf{R}_1\mathbf{R}_2 \cdot\cdot \mathbf{R}_n, \tag{33}$$

where $\mathbf{R}_T$ is the transfer matrix of the composite system from which the X-parameter matrix of the composite system $\mathbf{X}_T$ can be constructed.

VI. Results and Discussions

In order to test and validate the formulation, steady-state simulations were performed on a simple nonlinear link. First, the X-parameters for the circuit were generated using the ADS X-parameter generator [4]-[5]. Next the data was used by a simulation program that incorporated the solutions derived in the previous sections. Figure 3 shows the circuit schematic which consisted of a nonlinear driver (with cubic term), a microstrip line and a receiver. A 1-GHz sine wave was used to excite the system. Results from the X-parameter simulation program were found to correlate well with those of the standard ADS simulator as shown in the plots of Figure 3.

VII. Conclusion

This work demonstrated the use of the X-parameter/PHD formalism for the modeling and simulation of high-speed links. It assumes that the network driving conditions are such that the harmonic superposition principle is valid. A matrix formulation was presented and used to predict signal transmission in the frequency and time domains. Results showed good correlation with conventional circuit simulators. Thus, it is anticipated that X-parameters will be a critical tool for the performance prediction of high-speed communication systems involving nonlinear analog and digital components.

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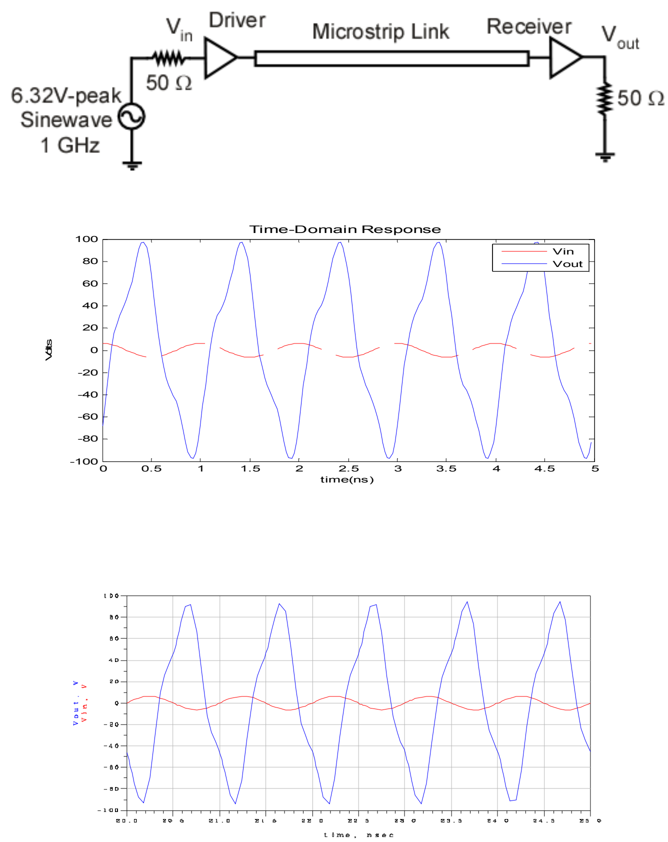

Figure 3. Circuit schematic (top), Matlab simulations using X- parameters (middle) and ADS simulations (bottom). Simulations show input (dashed) and output (solid) waveforms.

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