Blackbox Macromodel with S-Parameters and Fast Convolution

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Abstract
In this paper, the scattering parameters of blackbox multiport networks are pre-processed in the frequency domain to satisfy causality. Next, they are approximated to yield weighted delta functions in the time domain thus allowing the fast simulation. The resulting procedure leads to a robust, accurate and efficient macromodel generation scheme.

Introduction
Multiport networks characterized by sampled data or blackbox networks have become more frequent in the analysis or design of high-frequency or high-speed circuits. These systems offer significant challenges in the ability to accurately represent their behavior over a wide frequency range. In macromodel analysis, a complex network is described in terms of its terminal transfer functions in the frequency domain. This description consists of discrete data points obtained from a network analyzer or a full-wave field solver. A solution for the time-domain response can always be obtained by using convolution with the excitation provided at its terminals; however, this approach is computationally inefficient and becomes prohibitive for large networks. To circumvent the time-consuming convolution calculations, curve fitting techniques are used to approximate the blackbox data into a rational function in terms of its poles and residues. These poles and residues are next used in a time-domain recursive convolution algorithm that is far more computationally efficient than a direct convolution. Several methods exist in the literature that describe the generation of these macromodels. These include methods based on rational approximation, Padé synthesis and asymptotic waveform evaluation [1]-[2]. More recently, these macromodel efforts have progressively converged to the vector fitting method [3] which is today the method of choice for approximating transfer functions of a passive network. Several enhancements of the vector fitting method have been proposed that aim at improving the accuracy, bandwidth and computational complexity of the fitting process [4].

The extraction of poles and residues of a blackbox network can be tedious and cumbersome. For example, a typical serial link may require about 800 poles to accurately approximate its transfer functions. Such a large order in the rational function approximation can nullify the use of a recursive convolution. Furthermore, the poles and residues generated by the curve fitting process do not necessarily lead to a passive system. Consequently, a passivity enforcement scheme must be imposed into the frequency domain representation before performing the time domain simulation [5]-[6].

Commercial standard blackbox macromodels must be robust. In addition, they must satisfy the attributes of passivity, causality and realness. In this paper, we propose a robust macromodeling scheme for the simulation of passive multiport networks. A delta-function convolution method alleviates the computational burden of the time domain simulation, moreover, frequency-domain pre-processing insures well-behaved properties of the transfer functions.

Time-Domain Convolution with S-Parameters
In the frequency domain, the n-port linear blackbox S parameter formulation reads (see Figure 1)

$$B = SA$$

(1)

Where A and B are vectors of dimension $n$ representing the incident and reflected waves respectively. $S$ is an $n \times n$ matrix containing the S-parameter transfer functions.

In the time domain, the scattering parameter formulation reads

$$b(t) = s(t)^*a(t)$$

(2)

where $*$ indicates a convolution operation which is defined as

$$s(t)^*a(t) = \int_{-\infty}^{\infty} s(t-\tau)a(\tau)d\tau$$

(3)

When the time variable is discretized such that the current time variable $t$ is represented as $t = Ti$, the convolution becomes

$$s(t)^*a(t) = s(t)a(M\Delta t) + \sum_{\tau=0}^{M} s(\tau)a(M-\tau)\Delta t$$

(4)
where $\Delta t$ is the time step, $M$ is the integer index associated with the current time. The time-domain scattering parameter formulation then reads

$$b(t) = s'(0)\alpha(t) + H(t) \tag{5}$$

where $s'(0) = s(0)\Delta t$, and where $H(t)$ is the history of the network defined as:

$$H(t) = H(M) = \sum_{\tau=0}^{\Delta t} s(\tau)\alpha(t-\tau)\Delta t \tag{6}$$

In addition the incident and reflected waves can be related to the voltage and current measured at the terminals (Figure 2):

$$a(t) = \frac{1}{2}[v(t) + Z_0 i(t)] \tag{7}$$

and

$$b(t) = \frac{1}{2}[v(t) - Z_0 i(t)] \tag{8}$$

where $v(t)$ and $i(t)$ are the terminal voltage and current respectively. $Z_0$ is the reference matrix for scattering parameter. Equations (7) and (8) can be combined with S-parameter equations to yield

$$\frac{1}{2}[v(t) - Z_0 i(t)] = \frac{s'(0)}{2}[v(t) + Z_0 i(t)] + H(t) \tag{9}$$

After rearranging, we get

$$Z_0 i(t) + s'(0)Z_0 i(t) + 2H(t) = [1 - s'(0)]v(t) \tag{10}$$

or

$$[1 + s'(0)]Z_0 i(t) = [1 - s'(0)]v(t) - 2H(t) \tag{11}$$

from which

$$i(t) = Z_0^{-1}[1 + s'(0)]^{-1}[1 - s'(0)]v(t) - 2Z_0^{-1}[1 + s'(0)]^{-1}H(t) \tag{12}$$

which can be simplified as:

$$i(t) = Y_{\text{stomp}} v(t) - I_{\text{stomp}} \tag{13}$$

where

$$Y_{\text{stomp}} = Z_0^{-1}[1 + s'(0)]^{-1}[1 - s'(0)] \tag{14}$$

and

$$I_{\text{stomp}} = 2Z_0^{-1}[1 + s'(0)]^{-1}H(t). \tag{15}$$

The solution for $v(t)$ is found using (see Figure 2)

$$(Y_z + Y_{\text{stomp}})v(t) = I_z + I_{\text{stomp}}. \tag{16}$$

\section*{δ-Function Convolution}

Most of the computational burden in the time-domain simulation rests on the calculation of $H(t)$ in (7). To alleviate this problem, we assume that the discrete frequency-domain scattering parameter transfer functions can be described in the form

$$S(q) = \sum_{k=0}^{L} c_k e^{2\pi i q k} \tag{17}$$

in which the $c_k$'s and $k$'s are parameters to be determined. $L$ is the order of the approximation. With this representation, the associated time-domain function takes the form of a train of impulses whose weights are given by the $c_k$'s.

$$s(u) = \sum_{k=0}^{L} c_k \delta(u - k). \tag{18}$$

The $c_k$'s are obtained by taking the inverse discrete Fourier transform or IFFT of the frequency-domain transfer function. If the transfer functions are scattering parameters, most of these $c_k$'s will negligibly small and thus only a few ($L$) will need to be retained for the representation describer in $s(q)$. Convolution with an excitation function $a(p)$ becomes

$$H(p) = \left[ \sum_{k=0}^{L} c_k \delta(p - k) \right] * a(p) = \sum_{k=0}^{L} c_k a(p - k) \tag{19}$$

![Figure 3. Time-domain scattering parameter responses for a serial link showing the rapid decay of the function. Only the first 200 points from IFFT are shown.](image)

For scattering parameters $L$ is relatively small. For instance, the insertion loss scattering parameter of a backplane

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was measured on a network analyzer up to 6 GHz. When the data is processed through a 1601-point IFFT, only 20 points of the resulting time-domain sequence are larger than 1% of the maximum (absolute) value. This can also easily be observed by looking at plots of the impulse response (Figure 3). When the reference system is optimally chosen, the time domain scattering parameters die out quickly, leading to very few points in the delta-function sequence.

**DC Extrapolation**

Prior to performing the IFFT on the frequency-domain network parameters, we must insure that the data is suitable and satisfy conditions of passivity, causality and realizability. Most transfer functions used for blackbox multiport analysis are either obtained from network analyzer measurements or from full-wave electromagnetic solvers. Typically, the quantities obtained are scattering parameters which can be converted into other types of network parameters (Z or Y). Most network analyzers do not operate at very low frequencies and measured scattering parameters do not have DC data. Similarly, most full-wave field solvers suffer from serious inaccuracies at the lower frequencies and do not calculate DC transfer functions.

For IFFT processing, the DC information is critical and must represent the behavior of a realizable network. Therefore the DC extrapolation procedure becomes a necessary step.

On the Smith chart, S parameters follow the general pattern of growing or decaying clockwise-moving spirals with increasing frequency. Moreover, at DC, the S-parameters of a physical circuit value must be real and must lie on the horizontal axis of the Smith chart. With these considerations, we can assume a mathematical behavior described by

\[ S(f) = r_\infty e^{j\phi} + re^{j\phi} e^{-12\pi f / f_c} \]  

(20)

for the low-frequency behavior of an S parameter on the Smith chart. An algorithm can be devised to extract the values of \( r_\infty \), \( r \) and \( \phi \) using data points from the lowest frequencies [7]. Extrapolation of values for frequencies down to DC can then be achieved.

Figure 4 shows the Smith chart of \( S_{12} \) for a measured blackbox whose lowest measured frequency data point is at 2 GHz. The solid line represents the lowest frequency points from the data; the dashed lines indicate the points given by the DC extrapolation algorithm.

A DC extrapolation procedure does not guarantee physical realizability of the network described by (20). Additional constraints of causality and realizability must be imposed in conjunction with the IFFT procedure to ensure well-behaved properties of the associated time domain impulse functions. Techniques such as those described in [8] can be used to meet these requirements.

**Time-Domain Simulations**

Figure 5 shows the flow diagram describing the main steps in macromodel generation using our proposed method. The standard method using rational approximation in terms of poles and residues is also shown for comparison.

![Flow Diagram](image)

**TABLE 1: Run Time Comparison for Time-Domain Simulation**

<table>
<thead>
<tr>
<th>Method Type</th>
<th>4-Port - 399 Points (Max Freq: 20 GHz)</th>
<th>4-Port - 5000 Points (Max Freq: 50.0 GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Convolution (seconds)</td>
<td>0.14</td>
<td>45.5</td>
</tr>
<tr>
<td>( \delta )-function Convolution (seconds)</td>
<td>0.04</td>
<td>0.47</td>
</tr>
</tbody>
</table>
Finally, in order to evaluate the effectiveness of the DC extrapolation method, simulations were performed using a blackbox system for which the lowest frequency data was at 2 GHz. Waveforms using the extrapolation scheme were compared with those in which no DC extrapolation was performed. Results are shown in Figure 7 and indicate that substantial accuracy is gained from the method.

Conclusions

Frequency-domain pre-processing of blackbox data and simple δ-function representation of the time-domain scattering parameter impulse responses resulted in a robust, accurate and efficient macromodeling procedure. Time-domain scattering parameters are impulse responses that die out very rapidly because the traveling power waves are for the most part absorbed into the reference system. By utilizing this property, the time-domain convolution burden can be significantly reduced without significant alteration of the initial blackbox data.

References


