A Comparison of Two Latency Insertion Methods in Dependent Sources Applications

Ping Liu[1], Jilin Tan[2], Zhongyong Zhou[1]
PCB System Division
Cadence Design System, Inc
[1]Shanghai, China, [2]Chelmsford, MA USA
liuping@cadence.com

José Schutt-Ainé, Patrick Goh
Department of Electrical and Computer Engineering
University of Illinois at Urbana-Champaign
Urbana, IL USA
jschutt@emlab.uiuc.edu

Abstract— **This paper compares two different formulations of the latency insertion method (LIM) for the analysis of circuits with dependent sources. One is the scalar LIM and the other is the amplification matrix LIM. Numerical experiments demonstrate that the scalar LIM necessitates much smaller time step than the amplification matrix LIM when handling certain types of dependent sources indicating better performance of the latter technique without a sacrifice in accuracy.**

Keywords-latency insertion methods(LIM); scalar LIM; amplification matrix LIM; dependent sources

I. INTRODUCTION

With the increase in circuit density and I/O pin count in high-speed circuit systems, the simulation of large circuits such as power distribution networks (PDN) has become a serious challenge in the computer-aided design of the chip-package-board circuits. The latency insertion method (LIM) [1] has recently emerged as an efficient time-domain simulator for large-scaled networks. The method uses reactive latency in all branches and nodes of a circuit to generate update algorithms for the voltage and current quantities. The updating of branch currents and node voltages is performed in a leapfrog manner similar to the Yee algorithm used in the Finite-Difference Time-Domain (FDTD) method [2]. As a result, LIM has linear computational complexity and is thus substantially faster than the traditional matrix-vector product based methods such as SPICE[3].

In a chip-package-board network, non-linear devices represented by IBIS models are commonly encountered during simultaneous switching noise (SSN) analysis. IBIS models essentially consist of dependent sources and lumped RLGC circuits. Thus, simulation of dependent sources is very important in SSN analysis. Currently, two different types of LIM formulations exist for this analysis. One is the conventional LIM introduced in [1] and referred to as the scalar LIM, and the other is the amplification matrix LIM which was first presented in [4]. Both LIMs can support dependent source simulation. In this paper, we compare the two LIM algorithms and their efficiency in dealing with circuits containing dependent sources. The equations and the stability conditions of the explicit (forward Euler), semi-implicit (central difference) and fully-implicit (backward Euler) formula of the two LIMs are all presented and compared.

II. SCALAR LIM

LIM can analyse any arbitrary network in which one can define a branch as a connection between two nodes (excluding the ground reference node) through Thévenin and Norton transformations. Figure 1 shows a branch connecting nodes i and j has a resistor, an inductor and a voltage source in series. Node i combines a conductance, a capacitor and a current source in parallel. In the network, each branch must contain an inductance and each node must provide a capacitance path to ground. Otherwise, a small inductor or a small shunt capacitor should be inserted into the branch or connected to the node.

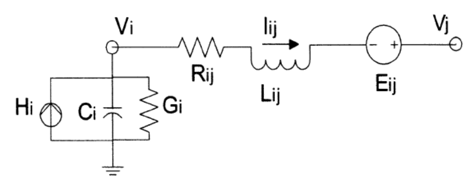

Figure 1. Normal linear branch and node

In order to solve the node voltages and the branch currents, LIM discretizes the time variables. There are three difference schemes which have different stability conditions. [5] In the standard LIM algorithm [1], the time-domain discrete equations for the linear elements in a branch linked a node are as follows. In the formulations, n represents the nth time step and Δt is the duration of one time step.

A. Explicit (Forward Euler)

$$I_{ij}^{n+1} = \left(1 - \frac{R_{ij}\Delta t}{L_{ij}}\right) I_{ij}^{n} + \frac{\Delta t}{L_{ij}}\left(V_i^{n+1/2} - V_j^{n+1/2} + E_{ij}^{n+1/2}\right) \quad (1)$$

$$V_i^{n+1/2} = \left(1 - \frac{\Delta t G_i}{C_i}\right) V_i^{n-1/2} - \frac{\Delta t}{C_i}(H_i^n - \sum_{k=1}^{Mi} I_{ik}^n) \quad (2)$$

In the explicit case, the resistor term RI is given by RI^n and the conductance term GV is given by $GV^{n-1/2}$. This is a first-order accurate scheme. It has stricter stability properties than FDTD. If the conductance $G = 0$, its stability condition is

$$\Delta t \le \sqrt{LC}\left[\sqrt{N^2+1} - N\right], \quad N = \frac{R}{4}\sqrt{\frac{C}{L}} \quad (3)$$

B. Semi-Implicit (Central Difference)

$$I_{ij}^{n+1} = \left(\frac{L_{ij}}{\Delta t} + \frac{R_{ij}}{2} \right)^{-1} \left[\left(\frac{L_{ij}}{\Delta t} - \frac{R_{ij}}{2} \right) I_{ij}^{n} + E_{ij}^{n+1/2} + V_i^{n+1/2} - V_j^{n+1/2} \right] \quad (4)$$

$$V_i^{n+1/2} = \left(\frac{C_i}{\Delta t} + \frac{G_i}{2} \right)^{-1} \left[\left(\frac{C_i}{\Delta t} - \frac{G_i}{2} \right) V_i^{n-1/2} - H_i^n - \sum_{k=1}^{Mi} I_{ik}^n \right] \quad (5)$$

In the semi-implicit case, the resistor term RI is given by $R\left(I^n + I^{n+1}\right)/2$ and the conductance term GV is given by $G\left(V^{n-1/2} + V^{n+1/2}\right)/2$. This is a second-order accurate scheme. It has the same stability properties as FDTD. If the conductance $G = 0$, its stability condition becomes

$$\Delta t \le \sqrt{LC} \quad (6)$$

C. Fully Implicit (Backward Euler)

$$I_{ij}^{n+1} = \left(1 + \frac{R_{ij}\Delta t}{L_{ij}} \right)^{-1} \left[I_{ij}^n + \frac{\Delta t}{L_{ij}} \left(V_i^{n+1/2} - V_j^{n+1/2} + E_{ij}^{n+1/2} \right) \right] \quad (7)$$

$$V_i^{n+1/2} = \left(\frac{C_i}{\Delta t} + G_i \right)^{-1} \left[\frac{C_i}{\Delta t} V_i^{n-1/2} - H_i^n - \sum_{k=1}^{Mi} I_{ik}^n \right] \quad (8)$$

In the fully implicit case, the resistive term RI is given by RI^{n+1} and the conductance term GV is given by $GV^{n+1/2}$. This has first-order accuracy. It has the more relaxed stability than FDTD. If the conductance $G = 0$, its stability condition is

$$\Delta t \le \sqrt{LC}\left[\sqrt{N^2+1} + N \right], \qquad N = \frac{R}{4}\sqrt{\frac{C}{L}} \quad (9)$$

The scalar LIM simulation is done by alternate "leapfrog" updates of branch-currents and node-voltages in the time domain according to the current and voltage equations.

III. AMPLIFICATION MATRIX LIM

If the current and voltage variables in Section II are written in a vector-matrix formulation, the scalar LIM becomes the amplification matrix LIM. Let us take the semi-implicit scalar LIM for example. Equation (4) and (5) can be written in a vector-matrix formulation as:

$$\vec{i}^{\,n+1} = \left(\frac{[L]}{\Delta t} + \frac{[R]}{2} \right)^{-1} \left[\left(\frac{[L]}{\Delta t} - \frac{[R]}{2} \right) \vec{i}^{\,n} + \vec{e}^{\,n+1/2} + [M]^T \vec{v}^{\,n+1/2} \right] \quad (10)$$

$$\vec{v}^{\,n+1/2} = \left(\frac{[C]}{\Delta t} + \frac{[G]}{2} \right)^{-1} \left[\left(\frac{[C]}{\Delta t} - \frac{[G]}{2} \right) \vec{v}^{\,n-1/2} + \vec{h}^{\,n} - [M]\vec{i}^{\,n} \right] \quad (11)$$

$\vec{i}^{\,n+1}$ is the branch current vector (dimension N_b) representing the currents at branches at time $n+1$, and $\vec{v}^{\,n+1/2}$ is the node voltage vector (dimension N_n) representing the voltages at the nodes at time $n+1/2$. [R] and [L] are the resistance and inductance matrices with dimension N_b by N_b respectively. [G] and [C] are the conductance and capacitance matrices with dimension N_n by N_n respectively. [M] is the $N_n \times N_b$ incidence matrix defined as follows

$M_{qp} = 1$ *if branch p is incident at node q and the current flows away from node q.*

$M_{qp} = -1$ *if branch p is incident at node q and the current flows into node q.*

$M_{qp} = 0$ *if branch p is not incident at node q.*

Explicit and fully implicit algorithms can be obtained in a similar manner.

One advantage of the amplification matrix LIM lies in its ability to accurately predict the time step ensuring the stability. To see this, we combine Eq. (10) and (11) together in a block matrix form and assume the sources are zero as follows:

$$\begin{bmatrix} \vec{v}^{\,n+1/2} \\ \vec{i}^{\,n+1} \end{bmatrix} = [A] \begin{bmatrix} \vec{v}^{\,n-1/2} \\ \vec{i}^{\,n} \end{bmatrix} \quad (12)$$

where

$$[A] = \begin{bmatrix} P^+P^- & -P^+[M] \\ Q^+[M]^T P^+P^- & Q^+Q^- - Q^+[M]^T P^+[M] \end{bmatrix} \quad (13)$$

in which we have defined

$$P^+ = \left(\frac{[C]}{\Delta t} + \frac{[G]}{2} \right)^{-1} \qquad P^- = \left(\frac{[C]}{\Delta t} - \frac{[G]}{2} \right)^{-1} \quad (14a)$$

$$Q^+ = \left(\frac{[L]}{\Delta t} + \frac{[R]}{2} \right)^{-1} \qquad Q^- = \left(\frac{[L]}{\Delta t} - \frac{[R]}{2} \right) \quad (14b)$$

[A] is called the amplification matrix since the voltages and the currents in the network will be amplified by [A] at each time step in the absence of sources. Therefore, to keep the simulation stable, the time step Δt should be chosen such that all the eigenvalues of the amplification matrix have magnitudes strictly less than 1. [6][9]

IV. TWO LIMS IN HANDLING DEPENDENT SOURCES

The scalar LIM and the amplification matrix LIM have some difference in handling the circuits with dependent sources. The difference lies on the updating equations. It is small but can cause large difference in performance of the two methods. To clarify this point, consider a simple circuit as example in which the fully implicit LIM is used for convenience.

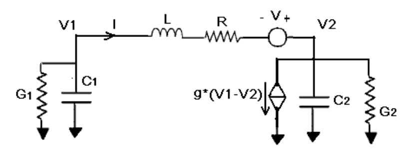

Figure 2. A circuit including a VCCS dependent source

Figure 2 shows a circuit including a voltage controlled current source (VCCS). Using the scalar LIM, we can create the node voltage and the branch current equations (15).

$$\begin{aligned} I_{vccs}^{n} &= g \cdot \left(V_1^{n+1/2} - V_2^{n+1/2} \right) \\ V_1^{n+1/2} &= \left(\frac{C_1}{\Delta t} + G_1 \right)^{-1} \left[\frac{C_1}{\Delta t} V_1^{n-1/2} - I^n \right] \\ V_2^{n+1/2} &= \left(\frac{C_2}{\Delta t} + G_2 \right)^{-1} \left[\frac{C_2}{\Delta t} V_2^{n-1/2} - \left(-I^n + I_{vccs}^{n} \right) \right] \end{aligned} \quad (15)$$

Obviously there is a paradox in the above equations. The controlled current at time *n* depends on the node voltage at time *n+1/2*. However the node voltage at time *n+1/2* also depends on the controlled current at time *n*. The scalar LIM cannot implement the closed loop in Eq(15). The solution is to replace I_{vccs}^{n} by I_{vccs}^{n-1} in the $V_2^{n+1/2}$ update equations. The above equations then become

$$
\begin{aligned}
&I_{vccs}^{n-1} = g\cdot\left(V_1^{n+1/2} - V_2^{n-1/2}\right) \\
&V_1^{n+1/2} = \left(\frac{C_1}{\Delta t}+G_1\right)^{-1}\left[\frac{C_1}{\Delta t}V_1^{n-1/2} - I^n\right] \\
&V_2^{n+1/2} = \left(\frac{C_2}{\Delta t}+G_2\right)^{-1}\left[\frac{C_2}{\Delta t}V_2^{n-1/2} - \left(-I^n + I_{vccs}^{n-1}\right)\right]
\end{aligned}
\tag{16}
$$

Eq.(16) introduces an approximation which leads to a decrease in accuracy.

The problem in Eq.(15) can be easily solved by the amplification matrix LIM without any approximation. Eq. (15) can be rewritten into a block matrix form as follows:

$$
\begin{pmatrix} V_1 \\ V_2 \end{pmatrix}^{n+\frac{1}{2}} = \begin{pmatrix} \dfrac{1}{1+\Delta t C_1^{-1}G_1} & 0 \\ \dfrac{-\Delta t C_2^{-1} g}{\left(1+\Delta t C_2^{-1}(G_2-g)\right)\left(1+\Delta t C_1^{-1}G_1\right)} & \dfrac{1}{\left(1+\Delta t C_2^{-1}(G_2-g)\right)} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}^{n-\frac{1}{2}} + \begin{pmatrix} \dfrac{-\Delta t C_1^{-1}}{1+\Delta t C_1^{-1}G_1} \\ \dfrac{\Delta t C_2^{-1}}{1+\Delta t C_2^{-1}(G_2-g)}\left[1+\dfrac{g\cdot\Delta t C_1^{-1}}{1+\Delta t C_1^{-1}G_1}\right] \end{pmatrix} I^n
\tag{17}
$$

Eqs (15) and (17) are entirely equivalent. However the scalar LIM cannot implement Eq. (15) without sacrificing the accuracy due to the closed loop. The amplification matrix LIM can implement Eq.(17) accurately.

For generality, the amplification matrix LIM formulations for general node and branch circuits including dependent sources are presented.

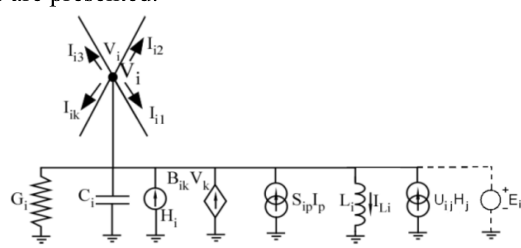

Figure 3. General node topology with dependent sources

Fig. 3 shows a node *i*'s topology including

- voltage controlled current source(VCCS) :

  - $B_{ik}V_k$: V_k is the controlling voltage at node k.

- current controlled current source(CCCS) :

  -$S_{ip}I_p$: I_p is the controlling current of branch p;

  -$U_{ij}H_j$: the controlling current H_j is an independent current source at a node.

- Node inductance L_i ; conductance G_i; capacitance C_i
- Independent current source H_i
- Independent voltage source E_i

Fig. 4 shows the topology of a branch between node *i* and *j* including

- voltage controlled voltage source(VCVS) :

  -$T_{ijk}V_k$: V_k is the controlling voltage of node k.

  -$W_{ijmn}E_{mn}$:the controlling voltage E_{mn} is an independent voltage source in a branch.

- current controlled voltage source(CCVS) :

  -$Z_{ijpq}I_{pq}$: I_{pq} is the controlling current of branch pq.

- Branch capacitance C_{ij}; inductance L_{ij}; resistance R_{ij}
- Independent voltage source E_{ij}
- Independent current source H_{mn}

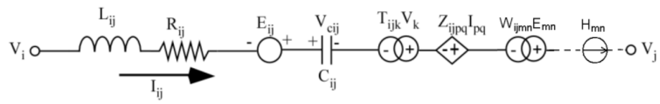

Figure 4. General branch topology with dependent sources

The block matrix formulation in semi-implicit LIM is

$$
\begin{pmatrix} \vec{V}^{n+\frac{1}{2}} \\ \vec{I}^{n+1} \end{pmatrix} = \bar{\bar{A}}' \begin{pmatrix} \vec{V}^{n-\frac{1}{2}} \\ \vec{I}^{n} \end{pmatrix} + \begin{pmatrix} 0 & \bar{\bar{P}}_+\bar{\bar{U}}' \\ \bar{\bar{Q}}_+\bar{\bar{W}}' & \bar{\bar{Q}}_+\overline{\overline{M^T}}'\bar{\bar{P}}_+\bar{\bar{U}}' \end{pmatrix} \begin{pmatrix} \vec{E}^{n+\frac{1}{2}} \\ \vec{H}^{n} \end{pmatrix} - \begin{pmatrix} 0 & \bar{\bar{P}}_+ \\ \bar{\bar{Q}}_+ & \bar{\bar{Q}}_+\overline{\overline{M^T}}'\bar{\bar{P}}_+ \end{pmatrix} \begin{pmatrix} \vec{V}_{C_N}^{\,n-\frac{1}{2}} \\ \vec{I}_{L_N}^{\,n-1} \end{pmatrix}
\tag{18}
$$

where

$$
\bar{\bar{A}}' = \begin{pmatrix} \bar{\bar{P}}_+\bar{\bar{P}}_- & -\bar{\bar{P}}_+\bar{\bar{M}}' \\ \bar{\bar{Q}}_+\overline{\overline{M^T}}'\bar{\bar{P}}_+\bar{\bar{P}}_- & \bar{\bar{Q}}_+\bar{\bar{Q}}_- - \bar{\bar{Q}}_+\overline{\overline{M^T}}'\bar{\bar{P}}_+\bar{\bar{M}}' \end{pmatrix} \qquad \bar{\bar{P}}_+ = \left(\frac{\bar{\bar{C}}}{\Delta t}+\frac{\bar{\bar{G}}'}{2}\right)^{-1}
$$

$$
\bar{\bar{P}}_- = \left(\frac{\bar{\bar{C}}}{\Delta t}-\frac{\bar{\bar{G}}'}{2}-\Delta t\cdot\overline{\overline{L_N}}^{-1}\right) \qquad \bar{\bar{Q}}_+ = \left(\frac{\bar{\bar{L}}}{\Delta t}+\frac{\bar{\bar{R}}'}{2}\right)^{-1} \qquad \bar{\bar{Q}}_- = \left(\frac{\bar{\bar{L}}}{\Delta t}-\frac{\bar{\bar{R}}'}{2}-\Delta t\cdot\overline{\overline{C_N}}^{-1}\right)
$$

$$
\bar{\bar{R}}' = \bar{\bar{R}} - \bar{\bar{Z}} \quad \overline{\overline{M^T}}' = \overline{\overline{M^T}} + \bar{\bar{T}} \quad \bar{\bar{W}}' = \bar{\bar{E}} + \bar{\bar{W}}
$$

$$
\bar{\bar{G}}' = \bar{\bar{G}} - \bar{\bar{B}} \quad \bar{\bar{M}}' = \bar{\bar{M}} - \bar{\bar{S}} \quad \bar{\bar{U}}' = \bar{\bar{E}} + \bar{\bar{U}}
$$

Some restrictions apply in the use of dependent sources for the amplification matrix LIM, namely:

- The controlling sources and the controlled sources must be in the same block;
- The controlling current cannot be the current through an element(G_i /C_i /L_i /E_i) at a node;
- If the controlled voltage source is at a node, it should be transformed into a controlled current source with a very small resistance inserted by a Norton transformation.
- If the controlled current source is in a branch, it should be changed to a controlled voltage source with a very large resistance using a Thévenin transformation.

We can use the new amplification matrix $\bar{\bar{A}}'$ to predict the stability of a time step Δt in the presence of dependent sources.

V. EXAMPLE

In this section we use a real case to compare the performance between the two LIMs in handling circuits with dependent sources. Fig. 5 shows a circuit including a voltage controlled voltage source (VCVS). The controlling coefficient *e* is 1.1. Since the controlled voltage source is at a node, it should be transformed into a controlled current source by the

Norton transformation. The circuit after transformation is shown in Fig.6. The inserted small resistor is 0.001 Ω.

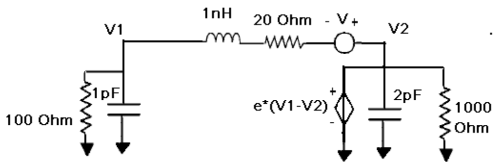

Figure 5. A circuit with a VCVS

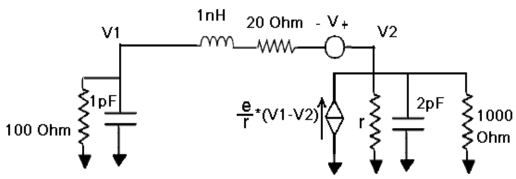

Figure 6. An equivalent circuit of the VCVS circuit

Table I is a maximum time step comparison between the scalar LIM and the Amplification-matrix LIM. The time step of the scalar LIM is obtained experimentally. The time step of the Amplification-matrix LIM can be predicted by the new amplification matrix $\overline{\overline{A}}'$ in Eq. (18). The update formulation we used in the scalar LIM is in the appendix. It is obvious the Amplification Matrix LIM can use much larger time-step if the semi-implicit formula or the backward Euler formula is adopted. Taking the semi-implicit LIM for example, the time-step of the scalar LIM is 3e-15 seconds while that of the amplification matrix LIM is 4e-11 seconds. Therefore, if the circuit includes dependent sources, it is best to use the amplification matrix LIM rather than the scalar LIM because the former has thousands of times higher performance than the latter.

TABLE I. MAXIMUM TIME STEP COMPARISON

Time-step (seconds)	Explicit (Forward Euler)	Semi-implicit (Central Difference)	Fully Implicit (Backward Euler)
Scalar LIM	1.9e-15	3e-15	4e-14
Amplification Matrix LIM	1.9e-15	4e-11	1e-10

VI. CONCLUSION

The conventional scalar LIM and the amplification matrix LIM were compared in terms of stability conditions. Their formulations and performances in handling circuits with dependent sources were also compared. This study shows that the amplification matrix LIM is superior by several orders of magnitude than the scalar matrix LIM and is a better option for circuits with dependent sources.

APPENDIX

Scalar method formulations for dependent source applications

In the Semi-implicit formulation and the Backward Euler formulation, when dependent sources are present, the voltage at a node depends on the voltage at a different node at the same time step. That is a closed loop, so in the scalar LIM, we use the values at the previous time step to replace those at the current time step as shown in the rectangular frames as follows.

A. *Explicit (Forward Euler)*

$$C_i\left(\frac{V_i^{n+1/2}-V_i^{n-1/2}}{\Delta t}\right)+G_iV_i^{n-1/2}-H_i^n+I_{Li}^n-B_{ik}V_k^{n-1/2}-S_{ip}I_p^n=-\sum_{j=1}^{M_i}I_{ij}^n \quad (19)$$

B. *Semi-Implicit (Central Difference)*

$$C_i\left(\frac{V_i^{n+1/2}-V_i^{n-1/2}}{\Delta t}\right)+\frac{G_i}{2}\left(V_i^{n+1/2}+V_i^{n-1/2}\right)-H_i^n$$
$$+I_{Li}^n-\frac{B_{ik}}{2}\left(V_k^{n+1/2}+V_k^{n-1/2}\right)-S_{ip}I_p^n=-\sum_{j=1}^{M_i}I_{ij}^n \quad (20)$$

$$C_i\left(\frac{V_i^{n+1/2}-V_i^{n-1/2}}{\Delta t}\right)+\frac{G_i}{2}\left(V_i^{n+1/2}+V_i^{n-1/2}\right)$$
$$-H_i^n+I_{Li}^n-\boxed{B_{ik}V_k^{n-1/2}}-S_{ip}I_p^n=-\sum_{j=1}^{M_i}I_{ij}^n \quad (21)$$

C. *Fully Implicit (Backward Euler)*

$$C_i\left(\frac{V_i^{n+1/2}-V_i^{n-1/2}}{\Delta t}\right)+G_iV_i^{n+1/2}-H_i^n+I_{Li}^n-B_{ik}V_k^{n+1/2}-S_{ip}I_p^n=-\sum_{j=1}^{M_i}I_{ij}^n \quad (22)$$

$$C_i\left(\frac{V_i^{n+1/2}-V_i^{n-1/2}}{\Delta t}\right)+G_iV_i^{n+1/2}-H_i^n+I_{Li}^n-\boxed{B_{ik}V_k^{n-1/2}}-S_{ip}I_p^n=-\sum_{j=1}^{M_i}I_{ij}^n \quad (23)$$

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