Modeling I/O Links With X Parameters

José E. Schutt-Aíné and Pavle Milosevic
Department of Electrical and Computer Engineering,
University of Illinois at Urbana-Champaign
Urbana, IL 61801

Wendemagegnehu T. Beyene
Research & Technology Development
Rambus Inc.
Los Altos, CA 94022
Outline

• Motivation
• S Parameters
• PHD Framework and X Parameters
  a. Definitions
  b. Properties
  c. Matrix Formulation
  d. Time-Domain Simulation
• Application to CMOS Inverter
• High-Speed Link Simulations
• Conclusions
Scattering Parameters

For a two-port

\[ V_1 = a_1 + b_1 \]
\[ b_1 = S_{11}a_1 + S_{12}a_2 \]
\[ I_1 = \frac{a_1 - b_1}{Z_0} \]
\[ b_2 = S_{21}a_1 + S_{22}a_2 \]
\[ I_2 = \frac{a_2 - b_2}{Z_0} \]

For a general N-port

\[ B = SA \]
\[ B_i = \sum_{j=1}^{N} S_{ij} A_j \]
\[ S_{ij} = \frac{B_i}{A_j} \text{ for } A_k = 0 \text{ if } k \neq j \text{ and } k = 1, \ldots, N \]

“…most successful behavioral models…”
Challenges in HS Links

High speed Serial channels are pushing the current limits of simulation. Models/Simulator need to handle current challenges

– Need to accurately handle very high data rates
– Simulate large number of bits to achieve low BER
– Non-linear blocks with time variant systems
– Model TX/RX equalization
– All types of jitter: (random, deterministic, etc.)
– Crosstalk, loss, dispersion, attenuation, etc…
– Handle and manage vendor specific device settings
– Clock data recovery (CDR) circuits

These cannot be accurately modeled with S parameters
X Parameters for SI

Objective

Adopt X parameters as the framework for high-speed channel design modeling and simulation.

Advantages

- Mathematically robust framework
- Can handle nonlinearities
- Instrument exists (NVNA)
- Blackbox format $\Rightarrow$ vendor IP protection
- Matrix format $\Rightarrow$ easy incorporation in CAD tools
- X Parameters are a superset of S parameters

Cascading X Parameters

GOAL: Simulate complete channel by combining X-parameter blocks from different sources into a single composite X matrix.

X-parameters of individual devices can be accurately cascaded within a harmonic balance simulator environment.
Nonlinear Vector Network Analyzer (NVNA)

NVNA instruments will gradually replace all VNAs
PHD Modeling

• Polyharmonic distortion (PHD) modeling is a frequency-domain modeling technique

• PHD model defines X parameters which form a superset of S parameters

• To construct PHD model, DUT is stimulated by a set of harmonically related discrete tones

• In stimulus, fundamental tone is dominant and higher-order harmonics are smaller
PHD Framework

• Signal is represented by a fundamental with harmonics

• Signals are periodic or narrowband modulated versions of a fundamental with harmonics

• Harmonic index: 0 for dc contribution, 1 for fundamental and 2 for second harmonic

• Power level, fundamental frequency can be varied to generate complete data for DUT
Excitation Design

Excitation 1
Excitation 2
Excitation 3
Excitation 4

Each excitation will generate response with fundamental and all harmonics
PHD Framework

\[ B_{1k} = F_{1k}(A_{11}, A_{12}, \ldots, A_{21}, A_{22}, \ldots) \]
\[ B_{2k} = F_{2k}(A_{11}, A_{12}, \ldots, A_{21}, A_{22}, \ldots) \]
Harmonic superposition principle is key to PHD model.

In many situations, there is only one dominant large-signal input component present. The harmonic frequency components are relatively small⇒ harmonic components can be superposed.

Harmonic superposition principle is key to PHD model.
X-Parameter Data File

TOP: FILE DESCRIPTION

! Created Fri Jul 30 07:44:48 2010

! Version = 2.0
! HB_MaxOrder = 25
! XParamMaxOrder = 12
! NumExtractedPorts = 3

! IDC_1=0   NumPts=1
! IDC_2=0   NumPts=1
! VDC_3=12   NumPts=1
! ZM_2_1=50   NumPts=1
! ZP_2_1=0   NumPts=1
! AN_1_1=100e-03(20.000000dBm)   NumPts=1
! fund_1=[100 Hz->1 GHz]   NumPts=4
BEGIN XParamData
% fund_1(real)  FV_1(real)  FV_2(real)  FI_3(real)  FB_1_1(complex)
% FB_1_2(complex)  FB_1_3(complex)  FB_1_4(complex)
% FB_1_7(complex)  FB_1_8(complex)  FB_1_9(complex)
% FB_1_12(complex)  FB_1_1(complex)  FB_2_2(complex)
% FB_2_5(complex)  FB_2_6(complex)  FB_2_7(complex)
% FB_2_10(complex)  FB_2_11(complex)  FB_2_12(complex)
% T_1_1_1_1(complex)  S_1_2_1_1(complex)  T_1_2_1_1(complex)
% S_1_4_1_1(complex)  T_1_4_1_1(complex)  S_1_5_1_1(complex)
% T_1_6_1_1(complex)  S_1_7_1_1(complex)  T_1_7_1_1(complex)
% S_1_9_1_1(complex)  T_1_9_1_1(complex)  S_1_10_1_1(complex)
% T_1_11_1_1(complex)  S_1_12_1_1(complex)  T_1_12_1_1(complex)
% T_2_1_1_1(complex)  S_2_2_1_1(complex)  T_2_2_1_1(complex)
% S_2_4_1_1(complex)  T_2_4_1_1(complex)  S_2_5_1_1(complex)
% T_2_6_1_1(complex)  S_2_7_1_1(complex)  T_2_7_1_1(complex)
% S_2_9_1_1(complex)  T_2_9_1_1(complex)  S_2_10_1_1(complex)
### X-Parameter Data File

#### BOTTOM: DATA LISTING

<table>
<thead>
<tr>
<th>X-Parameter Data File</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data is measured or generated from a harmonic balance simulator</td>
<td>Data file can be very large</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>0.903921</td>
<td>0.0263984</td>
</tr>
<tr>
<td>0.316228</td>
<td>-5.41159e-09</td>
</tr>
<tr>
<td>-5.8503e-16</td>
<td>-4.19864e-10</td>
</tr>
<tr>
<td>-6.37642e-16</td>
<td>-1.6748e-10</td>
</tr>
<tr>
<td>-1.25093e-15</td>
<td>-7.91128e-16</td>
</tr>
<tr>
<td>-1.51261e-10</td>
<td>1.93535e-17</td>
</tr>
<tr>
<td>-1.38032e-16</td>
<td>-2.09262e-10</td>
</tr>
<tr>
<td>0.107122</td>
<td>-5.52212e-08</td>
</tr>
<tr>
<td>-0.0081633</td>
<td>-2.40901e-08</td>
</tr>
<tr>
<td>-0.00739395</td>
<td>-1.21199e-08</td>
</tr>
<tr>
<td>-0.000530768</td>
<td>1.07836e-08</td>
</tr>
<tr>
<td>-0.00288533</td>
<td>-0.00230559</td>
</tr>
<tr>
<td>-2.56672e-09</td>
<td>-2.17127e-09</td>
</tr>
<tr>
<td>-3.25033e-15</td>
<td>-1.43757e-14</td>
</tr>
<tr>
<td>3.39598e-15</td>
<td>3.99868e-14</td>
</tr>
<tr>
<td>3.66098e-10</td>
<td>1.67366e-14</td>
</tr>
<tr>
<td>5.60242e-09</td>
<td>3.99868e-14</td>
</tr>
<tr>
<td>2.69755e-14</td>
<td>-6.60802e-10</td>
</tr>
<tr>
<td>3.99868e-14</td>
<td>3.99868e-14</td>
</tr>
</tbody>
</table>
X-Parameter Relationship

\[ b_{ik} = D_{ik} \left( |a_{11}| \right) P^k + \sum_{(j,l) \neq (1,1)} \left[ S_{ik,jl} \left( |a_{11}| \right) P^{k-l} a_{jl} + T_{ik,jl} \left( |a_{11}| \right) P^{k+l} a_{jl}^* \right] \]

- \( P \): Phase of \( a_{11} \)
- \( D_{ik} \): B-type X parameter
- \( S_{ik,jl} \): S-type X parameter
- \( T_{ik,jl} \): T-type X parameter
Index Convention

\[ S_{ik, jl} \]

\[ T_{ik, jl} \]

\[ a_{ik} \]

\[ b_{ik} \]
X Parameters of CMOS

S11,11 - Amplitude (dB)

0.5 GHz
1 GHz

T11,11 - Amplitude (dB)

0.5 GHz
1 GHz

S21,11 - Amplitude (dB)

0.5 GHz
1 GHz

T21,11 - Amplitude (dB)

0.5 GHz
1 GHz
X Parameters of CMOS

![Graphs showing X Parameters of CMOS](image-url)
Special Terms

• T-Type X Parameter
  - Spectral mapping is non-analytical
  - Real and imaginary parts in FD are treated differently
  - Even and odd parts in TD are treated differently
  - $T$ involves non-causal component of signal

• Phase Term $P$
  - $P$ is phase of large-signal excitation ($a_{11}$)
  - Contributions to B waves will depend on $P$
  - In measurements, system must be calibrated for phase
Handling Phase Term

\[ b_{ik} = D_{ik} \left( \left| a_{11} \right| \right) P^k + \sum_{(j,l) \neq (1,1)} \left[ S_{ik, jl} \left( \left| a_{11} \right| \right) P^{k-l} a_{jl} + T_{ik, jl} \left( \left| a_{11} \right| \right) P^{k+l} a_{jl}^* \right] \]

Multiply through by \( P^{-k} \)

\[ b_{ik} P^{-k} = D_{ik} \left( \left| a_{11} \right| \right) + \sum_{(j,l) \neq (1,1)} \left[ S_{ik, jl} \left( \left| a_{11} \right| \right) P^{-l} a_{jl} + T_{ik, jl} \left( \left| a_{11} \right| \right) P^{+l} a_{jl}^* \right] \]

\[ P = e^{j\phi_1} \text{ where } \phi_{11} \text{ is the phase of } a_{11} \]

we can always express the relationship in terms of modified power wave variables

\[ \overline{b}_{ik} = D_{ik} \left( \left| a_{11} \right| \right) + \sum_{(j,l) \neq (1,1)} \left[ S_{ik, jl} \left( \left| a_{11} \right| \right) \overline{a}_{jl} + T_{ik, jl} \left( \left| a_{11} \right| \right) \overline{a}_{jl}^* \right] \]

where \( \overline{b}_{ik} = b_{ik} P^{-k} \) and \( \overline{a}_{ik} = a_{ik} P^{-k} \)
Handling R&I Components

Because of non-analytical nature of spectral mapping, real and imaginary component interactions must be accounted for separately.

\[
\begin{pmatrix}
    b_r \\
    b_i
\end{pmatrix}
= \begin{pmatrix}
    X_{rr} & X_{ri} \\
    X_{ir} & X_{ii}
\end{pmatrix}
\begin{pmatrix}
    a_r \\
    a_i
\end{pmatrix}
\]

where

\[
X_{rr} = (S_r + T_r), \quad X_{ri} = -(S_i - T_i) \\
X_{ir} = (S_i + T_i), \quad X_{ii} = (S_r - T_r)
\]
Handling Phase Term

Phase term can be accounted for by applying following transformations

\[
\begin{pmatrix}
  b_r \\ b_i
\end{pmatrix} = \begin{pmatrix}
  X_{rr} & X_{ri} \\ X_{ir} & X_{ii}
\end{pmatrix} \begin{pmatrix}
  a_r \\ a_i
\end{pmatrix}
\]

\[
\begin{pmatrix}
  \cos \theta_b & -\sin \theta_b \\ -\sin \theta_b & \cos \theta_b
\end{pmatrix} \begin{pmatrix}
  b'_r \\ b'_i
\end{pmatrix} = \begin{pmatrix}
  X_{rr} & X_{ri} \\ X_{ir} & X_{ii}
\end{pmatrix} \begin{pmatrix}
  \cos \theta_a & -\sin \theta_a \\ -\sin \theta_a & \cos \theta_a
\end{pmatrix} \begin{pmatrix}
  a'_r \\ a'_i
\end{pmatrix}
\]

in which

\[
\begin{pmatrix}
  b_r \\ b_i
\end{pmatrix} = \begin{pmatrix}
  \cos \theta_b & -\sin \theta_b \\ -\sin \theta_b & \cos \theta_b
\end{pmatrix} \begin{pmatrix}
  b'_r \\ b'_i
\end{pmatrix}
\]

\[
\begin{pmatrix}
  a_r \\ a_i
\end{pmatrix} = \begin{pmatrix}
  \cos \theta_a & -\sin \theta_a \\ -\sin \theta_a & \cos \theta_a
\end{pmatrix} \begin{pmatrix}
  a'_r \\ a'_i
\end{pmatrix}
\]
X Matrix Construction

- Separate real and imaginary components
- Account for real-imaginary interactions
- Account for harmonic-to-harmonic contributions
- Account for harmonic-to-DC contributions

Matrix size is $2mn \times 2mn$

$m$: number of harmonics

$n$: number of ports
Matrix Formulation*

We wish to use:

\[ b = Xa \]

\( a = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_p \\ a_n \end{pmatrix} \)

\( b = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_p \\ b_n \end{pmatrix} \)

- vector size is \(2m\)
- \( m \): number of harmonics
- \( n \): number of ports

(\text{real vectors})

\*DC term not included

---

*DCON 2011
Where Chipheads Connect
Matrix Formulation*

$$X = \begin{pmatrix}
X_{11} & X_{12} & \cdots & X_{1n} \\
X_{21} & X_{22} & & \\
& \cdots & & \\
X_{n1} & & & X_{nn}
\end{pmatrix}$$

$X_{pq}$

(matrix size is $2mn$ $2mn$

$m$: number of harmonics $n$: number of ports size: $2m$ $2m$

$X_{pq}$

(real matrix)

*DC term not included

$X^{(11)}_{pqrr}$ $X^{(11)}_{pqri}$ $X^{(12)}_{pqrr}$ $X^{(12)}_{pqri}$ $X^{(1m)}_{pqrr}$ $X^{(1m)}_{pqri}$

$X^{(11)}_{pqir}$ $X^{(11)}_{pqii}$ $X^{(12)}_{pqir}$ $X^{(12)}_{pqii}$ $X^{(1m)}_{pqir}$ $X^{(1m)}_{pqii}$

$X^{(21)}_{pqrr}$ $X^{(21)}_{pqri}$ $X^{(22)}_{pqrr}$ $X^{(22)}_{pqri}$ $X^{(2m)}_{pqrr}$ $X^{(2m)}_{pqri}$

$X^{(21)}_{pqir}$ $X^{(21)}_{pqii}$ $X^{(22)}_{pqir}$ $X^{(22)}_{pqii}$ $X^{(2m)}_{pqir}$ $X^{(2m)}_{pqii}$

$X^{(m1)}_{pqir}$ $X^{(m1)}_{pqii}$ $X^{(mm)}_{pqir}$ $X^{(mm)}_{pqii}$
**X Matrix for 2-Port System**

(2 harmonics)

\[
X = \begin{pmatrix}
X_{11rr}^{(11)} & X_{11ir}^{(11)} & X_{11rr}^{(12)} & X_{11ir}^{(12)} & X_{12rr}^{(11)} & X_{12ir}^{(11)} & X_{12rr}^{(12)} & X_{12ir}^{(12)} \\
X_{11ir}^{(11)} & X_{11ii}^{(11)} & X_{11ir}^{(11)} & X_{11ii}^{(11)} & X_{12ir}^{(11)} & X_{12ii}^{(11)} & X_{12ir}^{(12)} & X_{12ii}^{(12)} \\
X_{11ir}^{(12)} & X_{11ii}^{(12)} & X_{11ir}^{(12)} & X_{11ii}^{(12)} & X_{12ir}^{(12)} & X_{12ii}^{(12)} & X_{12ir}^{(21)} & X_{12ii}^{(21)} \\
X_{11rr}^{(11)} & X_{21rr}^{(11)} & X_{21ir}^{(11)} & X_{21ii}^{(11)} & X_{22rr}^{(11)} & X_{22ir}^{(11)} & X_{22rr}^{(12)} & X_{22ir}^{(12)} \\
X_{21ir}^{(11)} & X_{21ii}^{(11)} & X_{21ir}^{(12)} & X_{21ii}^{(12)} & X_{22ir}^{(11)} & X_{22ii}^{(11)} & X_{22ir}^{(12)} & X_{22ii}^{(12)} \\
X_{21ir}^{(12)} & X_{21ii}^{(12)} & X_{21ir}^{(21)} & X_{21ii}^{(21)} & X_{22ir}^{(21)} & X_{22ii}^{(21)} & X_{22ir}^{(22)} & X_{22ii}^{(22)} \\
X_{21rr}^{(11)} & X_{21ir}^{(21)} & X_{21rr}^{(21)} & X_{21ir}^{(21)} & X_{22rr}^{(21)} & X_{22ir}^{(21)} & X_{22rr}^{(22)} & X_{22ir}^{(22)} \\
X_{21ir}^{(21)} & X_{21ii}^{(21)} & X_{21ir}^{(22)} & X_{21ii}^{(22)} & X_{22ir}^{(21)} & X_{22ii}^{(21)} & X_{22ir}^{(22)} & X_{22ii}^{(22)} \\
\end{pmatrix}
\]

(Real matrix)

For instance, \(X_{21ri}^{(12)}\) is the contribution to the real part of the 1\(^{st}\) harmonic of the wave scattered at port 2 due to the imaginary part of the 2\(^{nd}\) harmonic of the wave incident port in port 1.

*DC term not included
## Polyharmonic Impedance

<table>
<thead>
<tr>
<th>Linear Impedance</th>
<th>Polyharmonic Impedance</th>
<th>Nonlinear Impedance</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Time invariant</td>
<td>- Time invariant</td>
<td>- Time variant</td>
</tr>
<tr>
<td>- Linear</td>
<td>- Linear</td>
<td>- Nonlinear</td>
</tr>
<tr>
<td>- Scalar</td>
<td>- Matrix</td>
<td>- Function</td>
</tr>
<tr>
<td>$V = ZI$</td>
<td>$[V(f)] = [Z(f)][I(f)]$</td>
<td>$V(t) = Z(I(t))$</td>
</tr>
<tr>
<td>FD &amp; TD</td>
<td>FD only</td>
<td></td>
</tr>
</tbody>
</table>

Model assumes that nonlinear effects are mild and are captured via harmonic superposition.
Polyharmonic Impedance

4-harmonic system

in frequency domain:

\[
\begin{bmatrix}
V^{(1)} \\
V^{(2)} \\
V^{(3)} \\
V^{(4)}
\end{bmatrix} =
\begin{bmatrix}
Z^{(11)} & Z^{(12)} & Z^{(13)} & Z^{(14)} \\
Z^{(21)} & Z^{(22)} & Z^{(23)} & Z^{(24)} \\
Z^{(31)} & Z^{(32)} & Z^{(33)} & Z^{(34)} \\
Z^{(41)} & Z^{(42)} & Z^{(43)} & Z^{(44)}
\end{bmatrix}
\begin{bmatrix}
I^{(1)} \\
I^{(2)} \\
I^{(3)} \\
I^{(4)}
\end{bmatrix}
\]

in time domain:

\[v(t) = v^{(1)}(t) + v^{(2)}(t) + v^{(3)}(t) + v^{(4)}(t)\]

\[i(t) = i^{(1)}(t) + i^{(2)}(t) + i^{(3)}(t) + i^{(4)}(t)\]
Polyharmonic Impedance

\[ Z_o : \text{Reference impedance matrix} \]

\[ Z : \text{Polyharmonic impedance matrix} \]

\[ V : \text{Voltage vector} \]

\[ I : \text{Current vector} \]

Describes interactions between harmonic components of voltage and current.

\[ Z = (1 + X)(1 - X)^{-1} Z_o \]

\[ V = ZI \]
Network Formulation

Scattered waves

\[ b = Xa \]

Termination equations

\[ a = Dv_g + \Gamma b \]

Wave Solution

\[ a = \left[ 1 - \Gamma X \right]^{-1} Dv_g \]

Voltage Solution

\[ v = (1 + X)a \]
Steady-State Simulations

- **X Parameter**
- **cubic term**
- **ADS**
CMOS Driver/Receiver Channel

- Generate X parameters for composite system
- Power level: 20 dBm, frequency: 1 GHz
- Construct X matrix
- Combine with terminations for simulation
CMOS Driver/Receiver - Harmonics

DC+Fundamental

Vin  Vout

3 Harmonics

Vin  Vout

8 Harmonics

Vin  Vout

12 Harmonics

Vin  Vout
Equalized Channel

- ADS model of Tx (non-linear) + backplane channel (linear)
- Rx is passive termination
- Uses a typical BSIM3 model of a 0.25um 2.5V CMOS process, provided in ADS
  - Note: modified nfet and pfet to remove all parasitic caps, in order to run at higher speed.

- System Block Diagram:
Channel Analysis

Impulse Response, BR=5Gbps, t_r=20ps
Channel: 40-inch FR4, Z0=50Ohm; terminated with ZL=50 Ohm and Ci=2pF

• Unequalized impulse response
  – Reveals 1-tap FIR at Tx will cancel most of ISI (m7)

• Equalized impulse response
  – FIR tap coefficient set to -1/3 (ratio of m6 and m7)
  – DC shift due to equalizer structure
Transmitter Structure

• Input signal $V_{src}$ expected:
  – Single-ended 2.5V NRZ, 5Gbps, $t_r=20\text{ps}$
• FIR filter: modified single-ended push-pull
  – Output signal obtained by voltage dividers
  – Resistor sizing sets tap coefficients and DC levels

---

[Diagram showing the transmitter structure with input signal $V_{src}$ and output signal $V_{near}$, including a main branch and FIR tap 1 with delay of 1UI = 200ps.]
Transmitter Structure
Transient Response

Unequalized

Equalized

X Parameter

ADS

[Graphs showing transient response for unequalized and equalized cases with time and volts axes]
Far-End Eye Diagrams

Unequalized

Equalized

12 dBm

14 dBm
Conclusions

X Parameters represent a powerful format for the exchange of nonlinear behavioral models for use in the analysis and design of high-speed links

Challenges Ahead

- Standardization from different levels of approximation
- Define protocols for X-parameter exchange
References


The authors thank Agilent Technologies Inc., for encouraging this work and providing the ADS X-parameter generation platform, especially Loren Betts, Steve Fulwider and Bill Wallace for fruitful discussions, insightful comments and helpful suggestions.

_X-parameters_ is a registered trademark of Agilent Technologies, Inc.