MIDTERM EXAM

ECE 351  March 6, 2017

Instructions: Write your name and network ID where indicated. This examination consists of 3 problems. This is an open-book and open-notes exam. Use 50 Ω as the reference impedance for all measurement systems.

<table>
<thead>
<tr>
<th>Prob. 1 (40 pts)</th>
<th>Prob. 2 (40 pts)</th>
<th>Prob. 3 (20 pts)</th>
<th>Total (100 pts)</th>
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\[
T = \frac{P_1 \left[1 - \sum L(1) + \sum L(2) - ... \right] + P_2 \left[1 - \sum L(1) + \sum L(2) - ... \right]}{1 - \sum L(1) + \sum L(2) - \sum L(3) + ...}
\]

Mason's non-touching loop rule:
1. The above figure shows an air slotted-line. Voltage standing-wave minima locations are shown by the numbers for two measurements performed at a frequency of 375 MHz. Find the load reflection coefficient.

**Solution**

Find wavelength: \( \lambda = \frac{c}{f} = \frac{3 \times 10^8}{375 \times 10^6} = 0.008 \text{ m} = 80 \text{ cm} \)

\[
d_{\text{min}} = (90 \text{ cm} - 75 \text{ cm}) = 15 \text{ cm} = \frac{3\lambda}{16}
\]

\[
VSWR = \frac{V_{\text{max}}}{V_{\text{min}}} = \frac{6}{2} = 3, \quad \Gamma_R = -\left( \frac{VSWR - 1}{VSWR + 1} \right) e^{2j\beta d_{\text{min}}}
\]

\[
\Gamma_R = -\left( \frac{5-1}{5+1} \right) e^{2j\frac{3\lambda}{16}} = -\frac{2}{3} e^{j\frac{3\pi}{4}} = -\frac{2}{3} e^{j135^\circ}
\]

or \( \Gamma_R = -\frac{2}{3} e^{j\frac{3\pi}{4}} = \frac{2}{3} e^{j\frac{3\pi}{4}} = \frac{2}{3} e^{-j\frac{\pi}{4}} = \frac{2}{3} e^{-j45^\circ} \)

or \( \Gamma_R = \frac{2}{3} \left( \frac{1}{\sqrt{2}} - j \frac{1}{\sqrt{2}} \right) = \frac{\sqrt{2}}{3} - j \frac{\sqrt{2}}{3} \)
2. For the transmission line shown below, write the scattering parameter matrix as measured on a 50-Ω network analyzer.

![Transmission Line Diagram](image.png)

\[ Z_{o1} = 75 \ \Omega \]

**Solution**

\[
S_{11} = \frac{(1 - X^2)\Gamma}{1 - X^2\Gamma^2} \quad \text{and} \quad S_{21} = \frac{(1 - \Gamma^2)X}{1 - X^2\Gamma^2}
\]

with \( \Gamma = \frac{Z_{o1} - Z_o}{Z_{o1} + Z_o} \) and \( X = e^{-\frac{j2\pi l}{\lambda}} \)

\[
X = e^{-\frac{j2\pi l}{\lambda}} = e^{-j2\pi} = 1 \Rightarrow X^2 = 1
\]

\[
\Gamma = \frac{75 - 50}{75 + 50} = \frac{25}{125} = \frac{1}{5}
\]

\[
S_{11} = \frac{(1-1)\Gamma}{1-\Gamma^2} = 0
\]

\[
S_{21} = \frac{(1-\Gamma^2)}{1-\Gamma^2} = 1
\]

\[
S = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}
\]
3. For the circuit shown below (lab student unknown), the transmission lines use air as dielectric. What is the lowest frequency for which $\Gamma_{in} = 0$? (Use $Z_0$ as your reference impedance).

**Solution**

$\Gamma_{in} = 0$ when shorted stub length = $\lambda/4$, or

$$\frac{\lambda}{4} = 0.75 \text{ cm} \Rightarrow \lambda = 3 \text{ cm}$$

$$f = \frac{c}{\lambda} = \frac{0.3 \times 10^9}{3 \times 10^{-2}} = 10 \text{ GHz}$$