ECE 451
Coupled Lines

Jose E. Schutt-Aine
Electrical & Computer Engineering
University of Illinois
jose@emlab.uiuc.edu
Crosstalk Noise

Signal Integrity

Crosstalk  Dispersion  Attenuation
Reflection  Distortion  Loss
Delta I Noise  Ground Bounce  Radiation
TEM PROPAGATION

\[ \Delta z \]

\[ \begin{align*}
I & \quad L \\
+ & \quad C \\
V & \quad C \\
- & \\
\end{align*} \]

\[ Z_1 \quad Z_0 \quad \beta \quad Z_2 \]

\[ V_s \]
Telegrapher’s Equations

\[
\frac{\partial V}{\partial z} = L \frac{\partial I}{\partial t}
\]
\[
\frac{\partial I}{\partial z} = C \frac{\partial V}{\partial t}
\]

**L**: Inductance per unit length.

**C**: Capacitance per unit length.
Crosstalk noise depends on termination
Crosstalk depends on signal rise time

$t_r = 1$ ns

$t_r = 7$ ns
Crosstalk depends on signal rise time

\[ t_r = 1 \text{ ns} \]

\[ t_r = 7 \text{ ns} \]
Crosstalk depends on signal rise time

$t_r = 1 \text{ ns}$

$t_r = 7 \text{ ns}$
Coupled Transmission Lines
Telegraphers Equations for Coupled Transmission Lines

Maxwellian Form

\[-\frac{\partial V_1}{\partial z} = L_{11} \frac{\partial I_1}{\partial t} + L_{12} \frac{\partial I_2}{\partial t}\]

\[-\frac{\partial V_2}{\partial z} = L_{21} \frac{\partial I_1}{\partial t} + L_{22} \frac{\partial I_2}{\partial t}\]

\[-\frac{\partial I_1}{\partial z} = C_{11} \frac{\partial V_1}{\partial t} + C_{12} \frac{\partial V_2}{\partial t}\]

\[-\frac{\partial I_2}{\partial z} = C_{21} \frac{\partial V_1}{\partial t} + C_{22} \frac{\partial V_2}{\partial t}\]
Telegraphers Equations for Coupled Transmission Lines

Physical form

\[
- \frac{\partial V_1}{\partial z} = L_s \frac{\partial I_1}{\partial t} + L_m \frac{\partial I_2}{\partial t}
\]

\[
- \frac{\partial V_2}{\partial z} = L_m \frac{\partial I_1}{\partial t} + L_s \frac{\partial I_2}{\partial t}
\]

\[
- \frac{\partial I_1}{\partial z} = C_s \frac{\partial V_1}{\partial t} + C_m \frac{\partial V_1}{\partial t} - C_m \frac{\partial V_2}{\partial t}
\]

\[
- \frac{\partial I_2}{\partial z} = -C_m \frac{\partial V_1}{\partial t} + C_m \frac{\partial V_2}{\partial t} + C_s \frac{\partial V_2}{\partial t}
\]
Relations Between Physical and Maxwellian Parameters (symmetric lines)

\[ L_{11} = L_{22} = L_s \]

\[ L_{12} = L_{21} = L_m \]

\[ C_{11} = C_{22} = C_s + C_m \]

\[ C_{12} = C_{21} = -C_m \]
Even Mode

\[-\frac{\partial V_e}{\partial z} = \left( L_{11} + L_{12} \right) \frac{\partial I_e}{\partial t} \]

\[-\frac{\partial I_e}{\partial z} = \left( C_{11} + C_{12} \right) \frac{\partial I_e}{\partial t} \]

Add voltage and current equations

\[ V_e : \text{Even mode voltage} \quad V_e = \frac{1}{2}(V_1 + V_2) \]

\[ I_e : \text{Even mode current} \quad I_e = \frac{1}{2}(I_1 + I_2) \]

\[ Z_e = \sqrt{\frac{L_{11} + L_{12}}{C_{11} + C_{12}}} = \sqrt{\frac{L_s + L_m}{C_s}} \]

Impedance

\[ v_e = \frac{1}{\sqrt{(L_{11} + L_{12})(C_{11} + C_{12})}} = \frac{1}{\sqrt{(L_s + L_m)C_s}} \]

Velocity
Odd Mode

\[-\frac{\partial V_d}{\partial z} = (L_{11} - L_{12}) \frac{\partial I_d}{\partial t}\]

\[-\frac{\partial I_d}{\partial z} = (C_{11} - C_{12}) \frac{\partial I_d}{\partial t}\]

\(V_d: \text{Odd mode voltage}\)

\(V_d = \frac{1}{2}(V_1 - V_2)\)

\(I_d: \text{Odd mode current}\)

\(I_d = \frac{1}{2}(I_1 - I_2)\)

\(Z_d = \sqrt{\frac{L_{11} - L_{12}}{C_{11} - C_{12}}} = \sqrt{\frac{L_s - L_m}{C_s + 2C_m}}\)

\(V_d = \frac{1}{\sqrt{(L_{11} - L_{12})(C_{11} - C_{12})}} = \frac{1}{\sqrt{(L_s - L_m)(C_s + 2C_m)}}\)
Mode Excitation

Even Mode Excitation

Odd Mode Excitation
PHYSICAL SIGNIFICANCE OF EVEN- AND ODD-MODE IMPEDANCES

* $Z_e$ and $Z_d$ are the wave resistance seen by the even and odd mode travelling signals respectively.

* The impedance of each line is no longer described by a single characteristic impedance; instead, we have

\[
V_1 = Z_{11} I_1 + Z_{12} I_2
\]
\[
V_2 = Z_{21} I_1 + Z_{22} I_2
\]
Definitions

**Even-Mode Impedance:** $Z_e$
Impedance seen by wave propagating through the coupled-line system when excitation is symmetric (1, 1).

**Odd-Mode Impedance:** $Z_d$
Impedance seen by wave propagating through the coupled-line system when excitation is anti-symmetric (1, -1).

**Common-Mode Impedance:** $Z_c = 0.5Z_e$
Impedance seen by a pair of line and a common return by a common signal.

**Differential Impedance:** $Z_{diff} = 2Z_d$
Impedance seen across a pair of lines by differential mode signal.
EVEN AND ODD-MODE IMPEDANCES

$Z_{11}, Z_{22}$ : Self Impedances

$Z_{12}, Z_{21}$ : Mutual Impedances

For symmetrical lines,

$Z_{11} = Z_{22}$ and $Z_{12} = Z_{21}$
EXAMPLE
(Microstrip)

Single Line
Dielectric height = 6 mils
Width = 8 mils

Coupled Lines
Height = 6 mils
Width = 8 mils
Spacing = 12 mils

\[ \varepsilon_r = 4.3 \]
\[ Z_s = 56.4 \, \Omega \]
\[ \varepsilon_r = 4.3 \]

\[ Z_e = 68.1 \, \Omega \quad Z_d = 40.8 \, \Omega \]
\[ Z_{11} = 54.4 \, \Omega \quad Z_{12} = 13.6 \, \Omega \]
Even Mode

\[
I_{tdr} = \left[ \frac{a_e(t,0)}{Z_e} + \frac{a_d(t,0)}{Z_d} \right] + \left[ \frac{a_e(t,0)}{Z_e} - \frac{a_d(t,0)}{Z_d} \right]
\]

\[
V_{tdr} = a_e(t,0) - a_d(t,0)
\]

\[
\frac{V_{tdr}}{I_{tdr}} = \frac{Z_e}{2}
\]

\[
Z_e = 2\left( \frac{1+\rho_e}{1-\rho_e} \right) Z_g
\]

\[
v_e = \frac{2l}{\tau_e}
\]
Odd Mode

\[ V_{tdr} = a_e(t,0) + a_d(t,0) - [a_e(t,0) - a_d(t,0)] = V_f + V_b \]

\[ I_{tdr} = \left[ \frac{a_e(t,0)}{Z_e} + \frac{a_d(t,0)}{Z_d} \right] \quad I_{tdr} = - \left[ \frac{a_e(t,0)}{Z_e} - \frac{a_d(t,0)}{Z_d} \right] \]

\[ a_e(t,0) = 0, \quad \frac{V_{tdr}}{I_{tdr}} = 2Z_d \]

\[ Z_d = \frac{1}{2} \left( 1 + \rho_d \right) Z_g, \quad V_d = \frac{2l}{\tau_d} \]
EXTRACT INDUCTANCE AND CAPACITANCE COEFFICIENTS

\[ L_s = \frac{1}{2} \left[ \frac{Z_e}{v_e} + \frac{Z_d}{v_d} \right] \]

\[ C_s = \frac{1}{Z_e v_e} \]

\[ L_m = \frac{1}{2} \left[ \frac{Z_e}{v_e} - \frac{Z_d}{v_d} \right] \]

\[ C_m = \frac{1}{2} \left[ \frac{1}{Z_e v_e} - \frac{1}{Z_d v_d} \right] \]

\[ Z_d < Z_s < Z_e \]
Measured even-mode impedance
Measured odd-mode impedance

Odd-Mode Impedance

- $Z_d (\Omega)$
- Spacing (mils)

- $h=3$ mils
- $h=5$ mils
- $h=7$ mils
- $h=10$ mils
- $h=14$ mils
- $h=21$ mils
Measured even-mode velocity

Even-Mode velocity

Spacing (mils) vs. Even-Mode velocity

- h=3 mils
- h=5 mils
- h=7 mils
- h=10 mils
- h=14 mils
- h=21 mils
Measured odd-mode velocity

Odd-Mode Velocity

Spacing (mils)

- h=3 mils
- h=5 mils
- h=7 mils
- h=10 mils
- h=14 mils
- h=21 mils
Measured mutual inductance

Mutual Inductance

L_m (nH/m)

Spacing (mils)

h=3 mils
h=5 mils
h=7 mils
h=10 mils
h=14 mils
h=21 mils
Measured mutual capacitance
Even & Odd Mode Impedances
Modal Velocities in Stripline and Microstrip

**Microstrip**: Inhomogeneous structure, odd and even-mode velocities must have different values.

**Stripline**: Homogeneous configuration, odd and even-mode velocities have approximately the same values.
Microstrip vs Stripline

**Microstrip (h = 8 mils)**
- w = 8 mils
- ε_r = 4.32
- \( L_s = 377 \text{ nH/m} \)
- \( C_s = 82 \text{ pF/m} \)
- \( L_m = 131 \text{ nH/m} \)
- \( C_m = 23 \text{ pF/m} \)
- \( v_e = 0.155 \text{ m/ns} \)
- \( v_d = 0.178 \text{ m/ns} \)

**Stripline (h = 16 mils)**
- w = 8 mils
- ε_r = 4.32
- \( L_s = 466 \text{ nH/m} \)
- \( C_s = 86 \text{ pF/m} \)
- \( L_m = 109 \text{ nH/m} \)
- \( C_m = 26 \text{ pF/m} \)
- \( v_e = 0.142 \text{ m/ns} \)
- \( v_d = 0.142 \text{ m/ns} \)
Microstrip vs Stripline

Sense line response at near end

![Graphs showing microstrip and stripline responses](image-url)
General Solution for Voltages

\[ V_1(z) = \left( A_e e^{j\omega z} v_e + B_e e^{j\omega z} v_e \right)_{even} + \left( A_d e^{j\omega z} v_d + B_d e^{j\omega z} v_d \right)_{odd} \]

\[ V_2(z) = \left( A_e e^{j\omega z} v_e + B_e e^{j\omega z} v_e \right)_{even} - \left( A_d e^{j\omega z} v_d - B_d e^{j\omega z} v_d \right)_{odd} \]
General Solution for Currents

\[ I_1(z) = \frac{1}{Z_e} \left[ A_e e^{-j\omega z} v_e - B e^{j\omega z} v_e \right] + \frac{1}{Z_d} \left[ A_d e^{-j\omega z} v_d - B_d e^{j\omega z} v_d \right] \]

\[ I_2(z) = \frac{1}{Z_e} \left[ A_e e^{-j\omega z} v_e - B e^{j\omega z} v_e \right] - \frac{1}{Z_d} \left[ A_d e^{-j\omega z} v_d - B_d e^{j\omega z} v_d \right] \]
Coupling of Modes (asymmetric load)

First reflection  Second reflection
Coupling of Modes
(symmetric load)

First reflection
Second reflection
Three-Line Microstrip

\[- \frac{\partial V_1}{\partial z} = L_{11} \frac{\partial I_1}{\partial t} + L_{12} \frac{\partial I_2}{\partial t} + L_{13} \frac{\partial I_3}{\partial t} \]

\[- \frac{\partial V_2}{\partial z} = L_{21} \frac{\partial I_1}{\partial t} + L_{22} \frac{\partial I_2}{\partial t} + L_{23} \frac{\partial I_3}{\partial t} \]

\[- \frac{\partial V_3}{\partial z} = L_{31} \frac{\partial I_1}{\partial t} + L_{32} \frac{\partial I_2}{\partial t} + L_{33} \frac{\partial I_3}{\partial t} \]

\[- \frac{\partial I_1}{\partial z} = C_{11} \frac{\partial V_1}{\partial t} + C_{12} \frac{\partial V_2}{\partial t} + C_{13} \frac{\partial V_3}{\partial t} \]

\[- \frac{\partial I_2}{\partial z} = C_{21} \frac{\partial V_1}{\partial t} + C_{22} \frac{\partial V_2}{\partial t} + C_{23} \frac{\partial V_3}{\partial t} \]

\[- \frac{\partial I_3}{\partial z} = C_{31} \frac{\partial V_1}{\partial t} + C_{32} \frac{\partial V_2}{\partial t} + C_{33} \frac{\partial V_3}{\partial t} \]
Three-Line – Alpha Mode

Subtract (1c) from (1a) and (2c) from (2a), we get

\[- \frac{\partial V^\alpha}{\partial z} = (L_{11} - L_{13}) \frac{\partial I^\alpha}{\partial t} \]
\[- \frac{\partial I^\alpha}{\partial z} = (C_{11} - C_{13}) \frac{\partial V^\alpha}{\partial t} \]

This defines the Alpha mode with:

\[V^\alpha = V_1 - V_3 \quad \text{and} \quad I^\alpha = I_1 - I_3\]

The wave impedance of that mode is:

\[Z^\alpha = \sqrt{\frac{L_{11} - L_{13}}{C_{11} - C_{13}}}\]

and the velocity is

\[u^\alpha = \frac{1}{\sqrt{(L_{11} - L_{13})(C_{11} - C_{13})}}\]
Three-Line – Modal Decomposition

In order to determine the next mode, assume that

\[ V_\beta = V_1 + \beta V_2 + V_3 \]
\[ I_\beta = I_1 + \beta I_2 + I_3 \]

\[- \frac{\partial V_\beta}{\partial z} = (L_{11} + \beta L_{21} + L_{31}) \frac{\partial I_1}{\partial t} + (L_{12} + \beta L_{22} + L_{32}) \frac{\partial I_2}{\partial t} + (L_{13} + \beta L_{23} + L_{33}) \frac{\partial I_3}{\partial t} \]

\[- \frac{\partial I_\beta}{\partial z} = (C_{11} + \beta C_{21} + C_{31}) \frac{\partial V_1}{\partial t} + (C_{12} + \beta C_{22} + C_{32}) \frac{\partial V_2}{\partial t} + (C_{13} + \beta C_{23} + C_{33}) \frac{\partial V_3}{\partial t} \]

By reciprocity \( L_{32} = L_{23}, L_{21} = L_{12}, L_{13} = L_{31} \)

By symmetry, \( L_{12} = L_{23} \)

Also by approximation, \( L_{22} \approx L_{11}, L_{11} + L_{13} \approx L_{11} \)
Three-Line – Modal Decomposition

\[- \frac{\partial V_\beta}{\partial z} = (L_{11} + \beta L_{12} + L_{13}) \left( \frac{\partial I_1}{\partial t} + \frac{\partial I_3}{\partial t} \right) + (2L_{12} + \beta L_{11}) \frac{\partial I_2}{\partial t}\]

In order to balance the right-hand side into \(I_\beta\), we need to have

\[(2L_{12} + \beta L_{11}) I_2 = \beta (L_{11} + \beta L_{12} + L_{13}) I_2 \approx \beta (L_{11} + \beta L_{12}) I_2\]

\[2L_{12} = \beta^2 L_{12}\]

or \(\beta = \pm \sqrt{2}\)

Therefore the other two modes are defined as

The Beta mode with \(\beta = \pm \sqrt{2}\)
Three-Line – Beta Mode

The Beta mode with

\[ V_\beta = V_1 + \sqrt{2}V_2 + V_3 \]

\[ I_\beta = I_1 + \sqrt{2}I_2 + I_3 \]

The characteristic impedance of the Beta mode is:

\[ Z_\beta = \sqrt{\frac{L_{11} + \sqrt{2}L_{12} + L_{13}}{C_{11} + \sqrt{2}C_{12} + C_{13}}} \]

and propagation velocity of the Beta mode is

\[ u_\beta = \frac{1}{\sqrt{(L_{11} + \sqrt{2}L_{12} + L_{13})(C_{11} + \sqrt{2}C_{12} + C_{13})}} \]
Three-Line – Delta Mode

The Delta mode is defined such that

\[ V_\delta = V_1 - \sqrt{2}V_2 + V_3 \]

\[ I_\delta = I_1 - \sqrt{2}I_2 + I_3 \]

The characteristic impedance of the Delta mode is

\[ Z_\delta = \sqrt{\frac{L_{11} - \sqrt{2}L_{12} + L_{13}}{C_{11} - \sqrt{2}C_{12} + C_{13}}} \]

The propagation velocity of the Delta mode is:

\[ u_\delta = \frac{1}{\sqrt{(L_{11} - \sqrt{2}L_{12} + L_{13})(C_{11} - \sqrt{2}C_{12} + C_{13})}} \]
Symmetric 3-Line Microstrip

In summary: we have 3 modes for the 3-line system

$\begin{bmatrix} 1 & 0 & -1 \\ 1 & \sqrt{2} & 1 \\ 1 & -\sqrt{2} & 1 \end{bmatrix}$

- **Alpha mode**
- **Beta mode***
- **Delta mode***

*neglecting coupling between nonadjacent lines*
### Coplanar Waveguide

**Dielectric Constant**

\[ \varepsilon_r = 4.3 \]

**Parameters**

- \( L(nH/m) = \begin{pmatrix} 346 & 162 & 67 \\ 152 & 683 & 152 \\ 67 & 162 & 346 \end{pmatrix} \)
- \( C(pF/m) = \begin{pmatrix} 113 & 17 & 5 \\ 16 & 53 & 16 \\ 5 & 17 & 113 \end{pmatrix} \)

**Field Matrices**

- \( E = \begin{pmatrix} 0.45 & 0.12 & 0.45 \\ 0.5 & 0 & -0.5 \\ -0.45 & 0.87 & -0.45 \end{pmatrix} \)
- \( H = \begin{pmatrix} 0.44 & 0.49 & 0.44 \\ 0.5 & 0 & -0.5 \\ -0.10 & 0.88 & -0.10 \end{pmatrix} \)
Coplanar Waveguide

\[ Z_m(\Omega) = \begin{pmatrix} 73 & 0 & 0 \\ 0 & 48 & 0 \\ 0 & 0 & 94 \end{pmatrix} \quad Z_c(\Omega) = \begin{pmatrix} 56 & 23 & 8 \\ 22 & 119 & 22 \\ 8 & 23 & 56 \end{pmatrix} \]

\[ v_p(m/ns) = \begin{pmatrix} 0.15 & 0 & 0 \\ 0 & 0.17 & 0 \\ 0 & 0 & 0.18 \end{pmatrix} \]
**Coplanar Waveguide**

\[ K(k) : \text{Complete Elliptic Integral of the first kind} \]

\[
k = \frac{S}{S + 2W}
\]

\[
K'(k) = K(k')
\]

\[
k' = (1 - k^2)^{1/2}
\]

\[
Z_{ocp} = \frac{30\pi}{\sqrt{\varepsilon_r + 1}} \frac{K'(k)}{K(k)} \quad \text{(ohm)}
\]

\[
\nu_{cp} = \left( \frac{2}{\varepsilon_r + 1} \right)^{1/2} c
\]
Coplanar Strips

\[ Z_{ocs} = \frac{120\pi K'(k)}{\sqrt{\varepsilon_r + 1}} \left( \frac{K(k)}{K(k)} \right) \text{ (ohm)} \]
## Qualitative Comparison

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Microstrip</th>
<th>Coplanar Wguide</th>
<th>Coplanar strips</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_{\text{eff}}^*$</td>
<td>~6.5</td>
<td>~5</td>
<td>~5</td>
</tr>
<tr>
<td>Power handling</td>
<td>High</td>
<td>Medium</td>
<td>Medium</td>
</tr>
<tr>
<td>Radiation loss</td>
<td>Low</td>
<td>Medium</td>
<td>Medium</td>
</tr>
<tr>
<td>Unloaded Q</td>
<td>High</td>
<td>Medium</td>
<td>Low or High</td>
</tr>
<tr>
<td>Dispersion</td>
<td>Small</td>
<td>Medium</td>
<td>Medium</td>
</tr>
<tr>
<td>Mounting (shunt)</td>
<td>Hard</td>
<td>Easy</td>
<td>Easy</td>
</tr>
<tr>
<td>Mounting (series)</td>
<td>Easy</td>
<td>Easy</td>
<td>Easy</td>
</tr>
</tbody>
</table>

* Assuming $\varepsilon_r=10$ and $h=0.025$ inch
V Transmission Line

dielectric

ground reference

signal strip

ground reference
Calculated values of the characteristic impedance for a single-line v-strip structure as a function of width-to-height ratio w/h. The relative dielectric constant is $\varepsilon_r = 2.55$. 
Calculated values of the effective relative dielectric constant for a single-line v-strip structure as a function of width-to-height ratio w/h. The relative dielectric constant is $\varepsilon_r = 2.55$. 

**V-Line – Effective Permittivity**
Three-Line – V Transmission Line

![Diagram of Three-Line – V Transmission Line]

- Region 1
- Region 2
- Region 3
- Region 4
- Region 5
- Region 6

- Signal strip
- Dielectric
- Ground surface

Variables:
- w
- s
- 2p
- h
# V-Line and Microstrip

## Microstrip

<table>
<thead>
<tr>
<th>Region</th>
<th>2p</th>
<th>w</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>609.74</td>
<td>113.46</td>
<td>41.79</td>
</tr>
<tr>
<td>2</td>
<td>113.54</td>
<td>607.67</td>
<td>113.54</td>
</tr>
<tr>
<td>3</td>
<td>41.79</td>
<td>113.46</td>
<td>609.74</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Region</th>
<th>L (nH/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>425.84</td>
</tr>
<tr>
<td>2</td>
<td>20.35</td>
</tr>
<tr>
<td>3</td>
<td>7.00</td>
</tr>
</tbody>
</table>

## V-line

<table>
<thead>
<tr>
<th>Region</th>
<th>L (nH/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>113.54</td>
</tr>
<tr>
<td>2</td>
<td>607.67</td>
</tr>
<tr>
<td>3</td>
<td>41.79</td>
</tr>
</tbody>
</table>

Comparison of the inductance and capacitance matrices between a three-line v-line and microstrip structures. The parameters are p/h = 0.8 mils, w/h = 0.6 and $\varepsilon_r = 4.0$.  

![Microstrip Diagram](image)
V-Line: Coupling Coefficients

Plot of mutual inductance (top) and mutual capacitance (bottom) versus spacing-to-height ratio for v-line and microstrip configurations. The parameters are $w/h = 0.24$, $\varepsilon_r = 4.0$. 
V-Line vs Microstrip: Coupling Coefficients

Plot of the coupling coefficient versus spacing-to-height ratio for v-line and microstrip configurations. The parameters are w/h = 0.24, εr = 4.0.
Advantages of V-Line

* Higher bandwidth
* Lower crosstalk
* Better transition

Propagation Function

- Microstrip
- V-60
- V-30
V-Line vs Microstrip: Insertion Loss